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Piles Under Dynamic Loads

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SYNOPSIS The paper deals with some of the more recent developments in pile dynamics. It reviews the progress in the analysis of single piles and pile groups, field as well as laboratory experiments and soil-pile-structure interaction. The influence of pile-soil interface is discussed and extensive references are given.

INTRODUCTION

Piles have been used for hundreds of years but the last twenty years or so have seen a remarkable increase in interest in pile dynamics. There are a few reasons for this: good sites which do not require piles are getting scarcer and thus piling is used more widely; new important areas of application have emerged, for example offshore towers and nuclear powerplants; piles have repeatedly failed in earthquakes or were damaged; and finally, dynamics of shallow foundations has reached a point of satisfactory understanding thus shifting research interests to less understood foundation types. The aim of the studies is to increase the safety of the piles and the structures they support and to better understand the interaction between the piles and the structures under both critical and operational conditions.

The damage to piles may result from a few causes such as vibration effects, liquefaction, and embankment movements. A comprehensive survey of pile damage during earthquakes in Japan was presented by Mizuno (1987) but damage to piles also occurred in the Alaska earthquake of 1964, the Mexico City earthquake of 1985 and the Loma Prieta earthquake of 1989.

Pile behavior is, of course, very complex and this might have lead Terzaghi and Peck (1967) to state that "...theoretical refinements in dealing with pile problems...are completely out of place and can be safely ignored".

Fortunately, not everybody got discouraged by this pessimistic evaluation and a number of analytical and numerical approaches to the analysis of pile dynamic behavior have been developed. These approaches provided a much sounder theoretical basis for pile design than the equivalent cantilever concept or other purely empirical methods which dominated the field for decades. Nevertheless, some differences between the various theoretical approaches exist and the experiments reported are sometimes inconclusive; also, some uncertainties are inevitable when applying an idealized theory to field conditions. Thus, it may be useful to review some of the approaches in order that we may examine the differences among them and summarize what can be learned from experiments and field observations.

There are different dynamic loads that can act on piles: earthquake forces, wave forces, wind forces, machine unbalances etc. Here, the emphasis is primarily on conditions relevant to earthquake loading. Dealt with are properties and behavior of single piles and pile groups, interaction with the cap, pile experiments, pile-structure interaction and a few other topics. The subject of pile dynamics received a comprehensive treatment in the state-of-the-art report by Tajimi (1977), covering developments up to 1977, and in a few special volumes, i.e. De Beer et al. (1977), O'Neill and Dobry (1980), Nogami (1987) and Prakash and Sharma (1990). A number of papers on piles were presented to this conference. These are listed together at the end of the References. Among the special areas of pile dynamics not considered here are integrity testing and pile driving. Recent data on these subjects can be found in Fellenius (1988). So many papers have been published on pile dynamics since Tajimi's (1977) state-of-the-art report that it is impossible to refer to all of them in this report of limited scope. The author trusts that the readers will understand this.

SINGLE PILES

The earliest systematic, theoretical studies of dynamic soil-pile interaction are due to Parmelee et al. (1964), Tajimi (1966), Penzien (1970), Novak (1974) and a few others. Parmelee (1964) and Penzien (1970) employed a non-linear discrete model and a static theory to describe the dynamic elastic stress and displacement fields. Tajimi (1966) used a linear viscoelastic stratum of the Kelvin-Voigt type to model the soil and in his analysis of the horizontal response neglected the vertical component of the soil motion. Novak (1974) assumed linearity and an elastic soil layer composed of independent infinitesimally thin horizontal layers extending to infinity.

The different approaches formulated and the data they yield are briefly discussed below.

Single Piles in Homogeneous Soil

The analytical approaches treat the interaction between the pile and soil, schematically depicted in Fig. 1, in terms of continuum mechanics. The problem is very difficult to solve, even for the idealistic assumptions of linear elasticity or viscoelasticity, homogeneous soils and the pile being welded to the soil. Thus, approximate procedures were formulated first. Tajimi's (1966) solution of the horizontal response of an endbearing pile in a homogeneous layer, the first of its kind, neglected the vertical component of the motion. In 1974, Novak formulated a very simple approach based on plane strain soil reactions, which can be interpreted as dynamic Winkler medium or a plane strain, complex transmitting boundary placed directly to the pile. This boundary is similar to the standard viscous boundary but is frequency dependent and complex, i.e. it has a stiffness part in addition to the damping part. This solution identified dimensionless parameters of the problem, yielded a number of design charts and tables for dynamic stiffness and damping of piles, and indicated the effect of the pile static load on the horizontal pile characteristics. Material damping was later included in closed form expressions for the soil reactions in Novak et al. (1978). The application of the same approach to vertical response of floating piles (Novak, 1977) indicated great sensitivity of the pile behavior to tip condition and showed that floating piles generate more radiation damping but less stiffness than endbearing piles. Torsional response was also examined in this way (Novak & Howell, 1977, 1978) and the importance of material damping for this vibration mode was demonstrated.

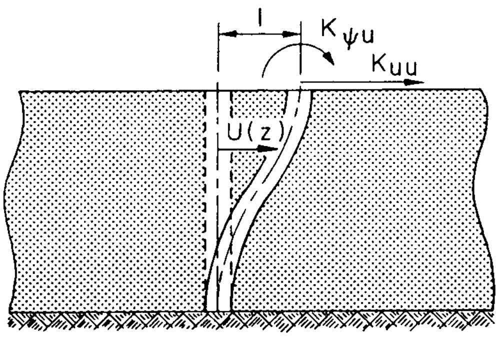

Figure 1 Schematic of soil-pile interaction

A somewhat more rigorous solution, similar to that of Tajimi (1966), was formulated by Nogami and Novak (1976) for the vertical response and for the horizontal response by Novak and Nogami (1977). These approximate solutions offered a basic insight into the behavior of the soil-pile system.

Much of the attention is focussed on the pile complex dynamic stiffnesses (impedance functions) because they have a strong influence on the response of pile supported buildings and structures. The impedance functions are defined as amplitudes of harmonic forces (or moments) that have to be applied to the pile head in order to generate a harmonic motion with a unit amplitude in the specified direction, as is schematically depicted in Fig. 1 for the case of horizontal impedance. The complex stiffnesses can be expressed in any of the following ways, i.e.

$$K = K_1 + iK_2 \tag{1a}$$

$$= k + i\omega c \tag{1b}$$

$$= \bar{k}(k' + i\omega c') \tag{1c}$$

in which K_1 and K_2 are the real and imaginary parts of the complex stiffness, respectively and $i = \sqrt{-1}$; $k = K_1$ = true stiffness, $c = K_2/\omega$ = coefficient of equivalent viscous damping, and ω = circular frequency; $\bar{k}$ = static stiffness and $k' = k/\bar{k}$, $c' = c/k'$ = dimensionless stiffness and damping constants. All the parameters in Eqs. 1 depend on frequency ω or the dimensionless frequency $a_o = r_o\omega/V_s$ where r_o = pile radius and V_s = soil shear wave velocity. An example of the horizontal impedance of endbearing piles is shown for two soil/pile stiffness ratios in Fig. 2. In this figure, v_c = primary wave velocity in the pile, L = pile length, and ν = Poisson's ratio; $D = 2\beta$ = soil material damping with β = soil material damping ratio and $\bar{\rho}$ = ratio of the soil specific mass to pile specific mass. The depressions visible in Fig. 2a practically disappear for higher soil

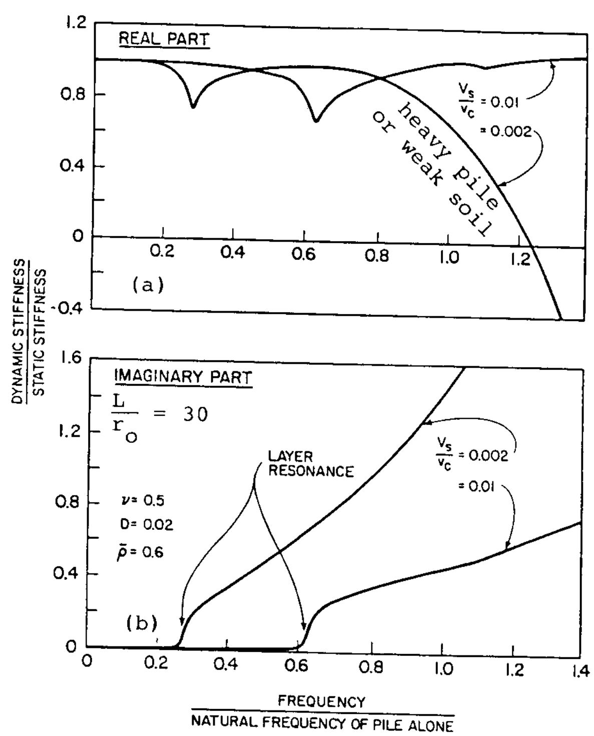

Figure 2 Horizontal impedance of endbearing pile for two soil/pile stiffness ratios (Novak & Nogami, 1977)

material damping as is schematically depicted in Fig. 3 in which the differences in impedance

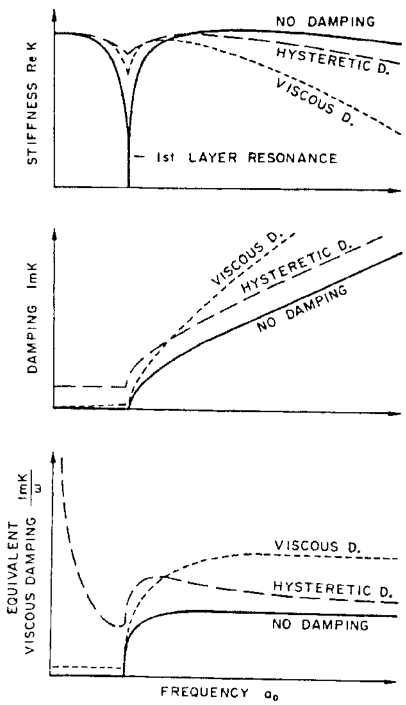

Figure 3 Impedances of endbearing pile for three cases of soil material damping

functions are emphasized for three cases of soil material damping, i.e. no material damping, hysteretic (frequency independent) material damping and viscous material damping. Hysteretic material damping is more realistic. For floating piles, the role of soil material damping is much smaller as the layer resonances are absent.

A few interesting features of the pile impedances follow from the theoretical solutions indicated in Figs. 2 and 3: pile dynamic stiffness varies little with frequency, except for very heavy piles or very weak soils for which it diminishes with frequency in a parabolic manner and can even become negative; for endbearing piles vibrating below the fundamental frequency of the soil layer, the geometric damping is absent because no progressive waves are generated in an elastic medium, just as with shallow foundations, leaving soil and pile material damping as the only sources of energy dissipation. Apart from this low frequency region, a fully embedded slender pile, not supporting any additional mass, is usually overdamped and consequently does not exhibit any marked resonance peak in dynamic tests.

More rigorous solutions, not neglecting one component of the motion, followed. For the horizontal response of an infinitely long pile, Kobori et al. (1977) obtained a solution in the form of infinite series of multiple integrals. More recent analyses based on the solution of the governing equations of a three dimensional continuum were formulated by Sen et al. (1985) and Pak and Jennings (1987). All vibration modes were investigated by Rajapakse and Shah (1987a,b, 1989). The latter authors evaluated the accuracy of some of the existing solutions and concluded that continuum models based on harmonic line loads may not be accurate enough and generated an extensive set of charts for impedances of floating piles.

The finite element solutions were formulated by Kuhlemeyer (1976, 1979a,b), Blaney et al. (1976), Wolf and von Arx (1978), Waas and Hartmann (1981), Sanchez-Salinero (1982) and others. Boundary element approaches were developed by Banerjee (1978), Banerjee and Sen (1987) and a few others. Ready to use charts and formulae were produced for homogeneous soils by Kuhlemeyer (1979a,b), Roesset (1980), Dobry et al. (1982), Novak and El Sharnouby (1983) and a few others. Thus, a considerable amount of data on piles in linear, homogeneous media is available. Some differences in these data exist but from the practical point of view, they agree reasonably well.

Since the pile dynamic stiffnesses for low frequencies are usually quite close to static stiffnesses, it may be useful to examine the differences in pile static stiffness. For axial loading such a comparison is shown in Fig. 4 in which $K' = Kr_o/A_pE_s$ is dimensionless static stiffness, $K = k$ is true static stiffness and A_p = pile cross-sectional area; E_p, E_s = pile and soil Young's modulus, respectively. Fig. 4 indicates that the individual authors' results differ substantially, particularly for flexible piles or very stiff soils, i.e. small E_p/E_s ratio; for endbearing piles even an illogical trend for the stiffness to increase with pile length, L, may be noticed (Fig. 4a). Inaccuracies of this type result primarily from the small number of elements used in pile discretization. In the El Sharnouby and Novak (1990) analysis, fifty elements or more were needed to eliminate the upward trend visible in Fig. 4a.

Single Piles in Nonhomogeneous Soil

Comparing the results of experiments with theoretical predictions repeatedly showed that if the soil is assumed to be homogeneous, both pile stiffness and damping can be grossly overestimated (e.g. Novak & Grigg, 1976; Novak & Sheta, 1982). An example of this is shown in Fig. 5 in which the theoretical response was calculated with two constant values of shear wave velocity: the value V_t, established from a wave propagation experiment (curve A) and a much lower value, $V_s = 0.26\ V_t$ backcalculated from measured static deflections (curve B). Such a reduced value yields a better estimate of pile stiffness (resonance frequency) but does not allow a satisfactory prediction of radiation damping and thus resonant amplitude. The reasons for the deficiencies of the theory based on the assumption of soil homogeneity are schematically depicted in Fig. 6. They are the variation of soil shear modulus with depth, particularly its reduction toward ground surface which results from the diminishing confining pressure, and pile separation from the soil or gapping. Single piles under horizontal loading, as in Fig. 5, are particularly sensitive to these two factors.

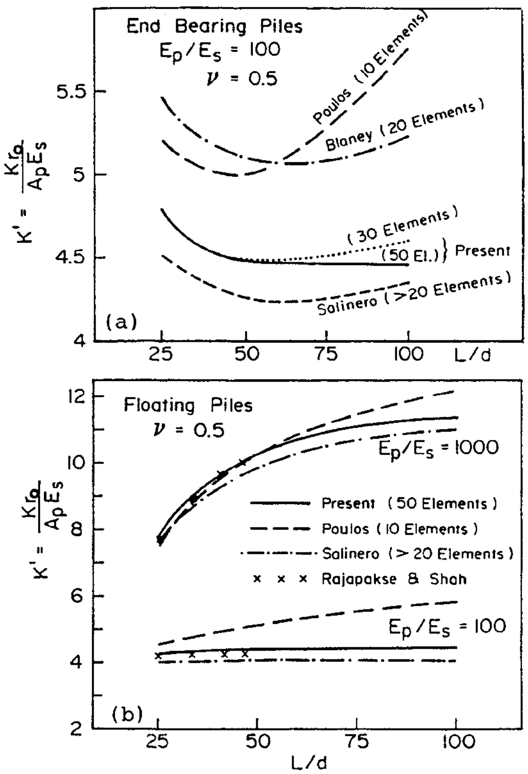

Figure 4 Comparison of static axial pile stiffness calculated by different authors for homogeneous soil: (a) - endbearing piles, (b) - floating piles (Present data by El Sharnouby and Novak, 1990)

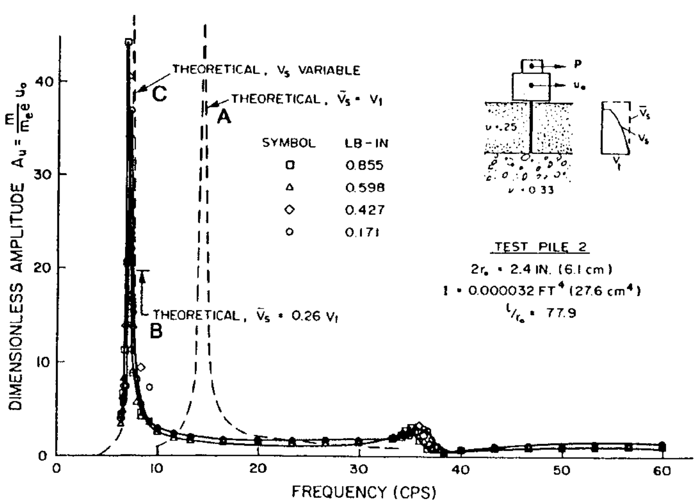

Figure 5 Comparison of experimental horizontal response of steel test pile with theoretical predictions (Novak and Sheta, 1982)

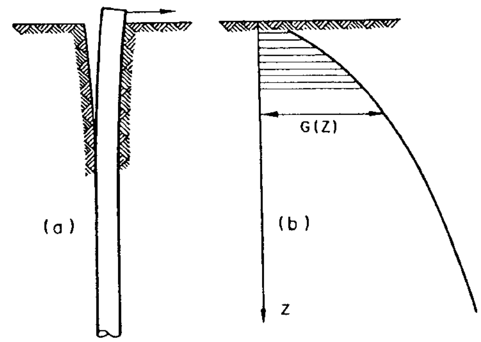

Figure 6 Schematic of pile separation and soil modulus reduction towards ground surface

Observations of this kind lead to the development of approaches better suited for nonhomogeneous soils. A significant improvement in the finite element model was formulated by Roesset and his co-workers (Blaney et al., 1976; Roesset & Angelides, 1979) who placed the consistent, frequency dependent boundary, derived by Kausel et al (1975), directly to the pile or outside the cylindrical finite element zone around the pile. This approach was then used by Krishnan et al. (1983) and by Gazetas (1984) in their extensive parametric studies.

Significant further progress was made by Kaynia (1982a,b) and Kaynia and Kausel (1982, 1990) who based their solution of piles in generally layered media on the formulation of displacement fields due to uniformly distributed forces on cylindrical surfaces (so called barrel load). (This solution will be discussed in more detail in the paragraph on pile groups.)

Banerjee and Sen (1987) presented a boundary element solution for piles embedded in a semi-infinite nonhomogeneous soil in which the soil modulus, E_s, varies linearly with depth, z. Banerjee and Sen's results suggest that, unlike in layered soils, the frequency variations of the impedance functions, normalized by static stiffness, are quite smooth and are affected very little by soil nonhomogeneity. The actual magnitude of the stiffness and damping diminishes with $E_s(0)$, however.

A few other methods suitable for linear generally layered media use a semi-analytical finite element approach. These methods treat the wave propagation in the horizontal direction analytically and in the vertical direction employ finite element idealization including auxiliary sublayers. The pile is modelled by beam elements. One of the advantages of this approach is that it may avoid the mathematical ill-conditioning resulting from the large magnitude of Lamé's constant, λ, for soil Poisson's ratio, ν, approaching 0.5. Solutions of this type were formulated by Tajimi and Shimomura (1976), Shimizu et al. (1977), Waas and Hartmann (1981, 1984) and Mizuhata and Kusakabe (1984).

An approximate analytical solution based on the extension of the Novak and Nogami (1977) approach was formulated for layered media by Takemiya and Yamada (1981).

A much simpler and very versatile solution, particularly well suited for high frequencies, was formulated by Novak and Aboul-Ella (1978a,b) who extended the plane strain approach to include layered media and incorporated it in the code PILAY. This code was used later by Novak and El Sharnouby (1983) to generate design charts and tables for parabolic soil profiles, as well as homogeneous ones. With this approach, and assuming a parabolic soil profile, with an allowance for pile separation in the form of a small free length, very satisfactory agreement with the theory was obtained as indicated by curve C in Fig. 5. Roesset et al. (1986) also found the plane strain approach to work very well for high frequencies. For very low frequencies, an adjustment to the plane strain soil reaction is made for the vertical and horizontal directions as discussed in Novak and El Sharnouby (1983) and implemented in the code PILAY. The plane strain approach works well for high frequencies because, in a layer, elastic waves tend to propagate more and more horizontally as the frequency increases, like in a wave guide.

The sensitivity of the response to pile separation and free length shows when evaluating most experiments. The prediction of the separation length is difficult and only empirical suggestions can be made at this time. For small amplitudes, Δ, El-Marsafawi et al. (1990) observed the following approximate relationship for pile separation length, L_s:

$$\frac{L_s}{d} = 260\,\frac{\Delta}{d}\,,\quad 0.001 \le \frac{\Delta}{d} \le 0.005 \qquad (2)$$

For larger displacements, a large separation length may be needed (Han and Novak, 1988). More data on the separation effect will be given in the paragraph on nonlinear response.

As for possible deviations of the theoretical assumptions from reality, pile deficiencies may also have a profound effect. This is shown by Wu et al. (1991) who, in their paper to this conference, examine the influence of pile necking using a combination of the BEM and FEM.

Radial nonhomogeneity

While the consideration of a free separation length in the analysis may produce the reduction in both pile stiffness and damping often observed in experiments, a better measure to this effect, or a complementary one, may be to account for soil nonhomogeneity in the radial direction. A simple way of doing this is to assume a weak, cylindrical boundary zone around the pile (Fig. 7). The zone is homogeneous and features a soil shear modulus, G_i, smaller than that of the outer zone and a larger material damping. The purpose of such a zone is to account in a very approximate way for soil nonlinearity in the region of the highest stresses, pile separation, slippage and other deficiencies of the pile-soil interface. Such a zone was proposed by Novak and Sheta (1980). In their plane strain solution, the mass of the boundary zone was neglected in

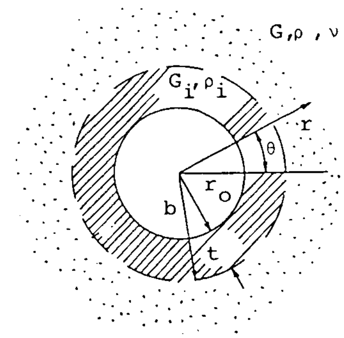

Figure 7 Cylindrical boundary zone around pile

order to prevent wave reflections from the fictitious interface between the cylindrical zone and the outer region. These reflections occur with nonzero weak zone mass, ρ_i, and result in undesirable undulations in both stiffness and damping of the composite medium. This is exemplified in Fig. 8 in which α and β are

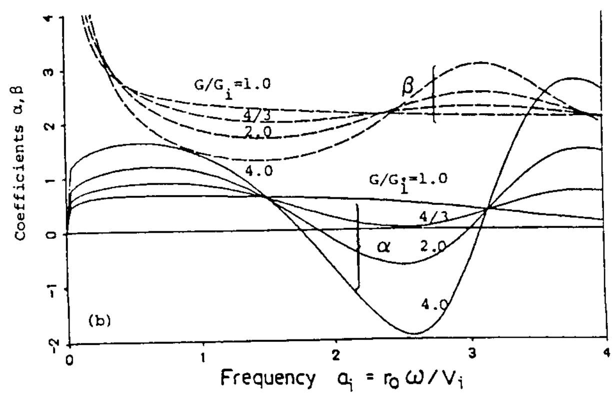

Figure 8 Dimensionless vertical impedances of composite medium with $\rho_i = \rho$ and $t/r_o = 1.0$ (soil damping ratio = 0.05)

nondimensional stiffness and damping constants of the composite medium respectively. These undulations can make the solution with $\rho_i \neq 0$ actually less suitable for practical applications (Novak and Han, 1990). The difficulty with wave reflections can be avoided by providing for a continuous transition of stresses from the inner zone to the outer region. Such a solution was explored by Lakshmanan and Minai (1981), Dotson and Veletsos (1990) and Mizuhata and Kusakabe (1984). The latter authors found that even with the weak zone, the experimental resonance amplitude measured on a 43.2 m long pile was five times larger than the theoretical value while the resonance frequency was predicted quite well. This is consistent with other observations and indicative of the need to account for pile separation.

Wolf and Weber (1986) conducted a more rigorous study of the effect of soil tension exclusion, also assuming the circular cavity in the unbounded thin layer (plane strain). They found that soil separation hardly affects horizontal stiffness, k_h, but reduces damping, c_h, by more than 50 per cent (Fig. 9b), a result quite similar to that of Novak and Sheta (1980). In addition, if shear is eliminated and hence slipping is allowed in the zone of contact, stiffness is also strongly reduced (Fig. 9c). In Fig. 9, the linear case (a) indicates the analysis with tension allowed. The size of the contact area appears to be of little effect. Many other authors studied the interface behavior. Among the more recent ones are Mamoon (1990) and He (1990). However, when applying the various plane strain approaches to the interface, the variation with depth is a problem for which very little guidance is available.

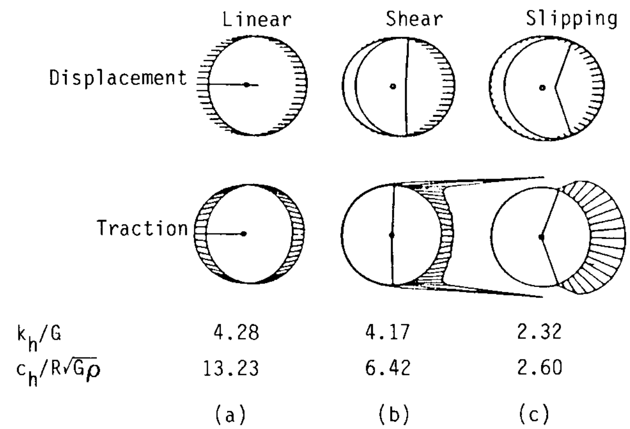

Figure 9 Effect of elimination of tension in separation zone (Wolf & Weber 1986; a_o=0.629, ν=0.48)

Recognizing the separation effect and allowing for it in an approximate way, a reasonable agreement between the theoretical results and experiments can be obtained. This is exemplified in Fig. 10 comparing the theoretical and experimental responses of a concrete pile 7.5 m in length and 0.32 m in dia. The soil was multilayered and a cylindrical weak zone was assumed when calculating the response using the code DYNA3. In this code, the weak zone is analyzed as massless but its mass is added to that of the pile in full or in part. Similar tests and comparisons were reported by Gle (1981), Woods (1984) and a number of others.

Nonlinear Response of Single Piles

The theories discussed thus far are essentially linear and thus quite adequate for small displacements. At large displacements, piles behave in a nonlinear fashion because of soil nonlinearity at high strain, pile separation (gapping), slippage and friction. To incorporate these factors into a continuum theory is extremely difficult and therefore, lumped mass models are most often used when nonlinear analysis is required. Such models, employed by

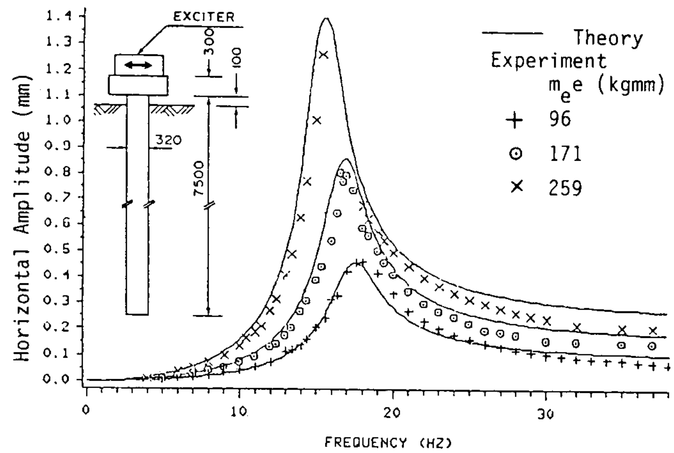

Figure 10 Theoretical and experimental horizontal response of concrete pile for three levels of harmonic excitation (El Marsafawi et al., 1990)

Penzien (1970), Matlock et al. (1978, 1980) and a number of others, feature nonlinear springs, nonlinear dampers, gaps and Coulomb friction blocks. The combination of these elements makes it possible to generate a variety of nonlinear force-displacement relationships. An example of the lumped mass model, formulated by Matlock et al. (1978, 1980) and incorporated in the code SPASM, is shown in Fig. 11. Models of this type are very versatile but it is difficult to relate the characteristics of the discrete elements to standard geotechnical parameters of soil. To help overcome this difficulty, various nonlinear

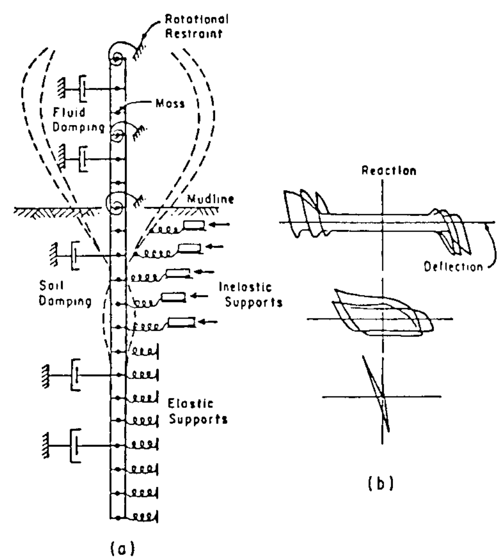

Figure 11 (a) Nonlinear lumped mass model of pile, (b) Observed cyclic reaction-deflection characteristics (Matlock et al., 1978) soil resistance-deflection relationships known as p-y curves and t-x curves have been recommended in the literature. For applications in offshore structures, the American Petroleum Institute (1986) specifies the p-y curves for clay as well as sand making a difference between static loading and cyclic loading. Extensive data on the p-y curves and nonlinear pile response were obtained by Yan (1990) using model piles and the hydraulic gradient similitude method to reproduce prototype conditions. An example of Yan's results is shown in Fig. 12. Notice the narrowing and partial linearization of the hysteresis loop with the number of cycles; this trend increases with depth.

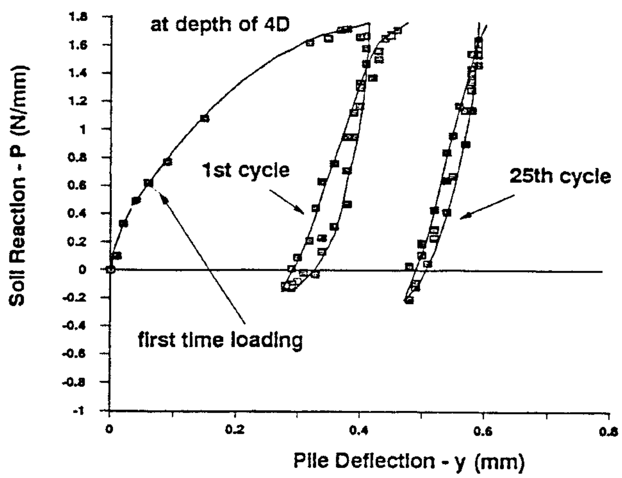

Figure 12 Example of p-y curve under cyclic loading (Yan, 1990)

Cyclic loading is defined as repetitive loading with very low frequency so that no significant inertia forces and radiation damping arise. It provides basic insight into the material degradation due to soil plasticity and mechanic degradation due to gapping associated with large displacements. Many studies were devoted to this subject but only a few may be mentioned here. Trochanis et al. (1988) found theoretically a dramatic decrease in pile stiffness due to gapping. Morrison and Reese (1988) conducted extensive full scale investigation of piles and pile groups. To this conference, Purkayastha and Dey (1991) report on their experimental study of the degradation of vertical stiffness. Summarizing their observations, Swan and Poulos (1982) postulate that during cyclic lateral loading the two forms of degradation lead to the increase in pile deflection and bending stresses; but if this degradation stabilizes, the pile is said to "shakedown" to a state of permanent strains and residual stresses and will react elastically to any further cyclic loading of the same intensity. When the pile does not stabilize into an elastic or inelastic response, the pile deflections continue to increase and incremental collapse may result. The two situations are depicted in Fig. 13.

The shakedown phenomenon is favourable from the

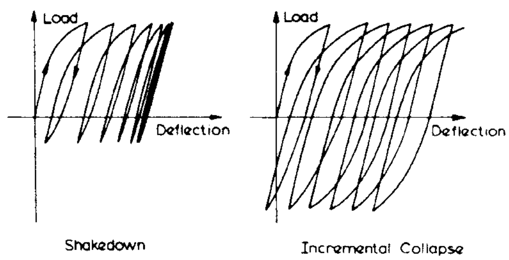

Figure 13 Pile stabilization (shakedown) and incremental collapse under cyclic loading with constant amplitude (Swane & Poulos, 1982)

point of view of the applicability of the various linear theories for dynamic response analysis. It explains why, with adequate adjustments particularly for pile separation, such theories may give reasonable results, as in Fig. 10, even in cases where overall strong nonlinearity of the response is clearly manifested.

Under vertical, steady state vibration, a similar stabilization and partial linearization takes place. Figure 14 shows the vertical displacement

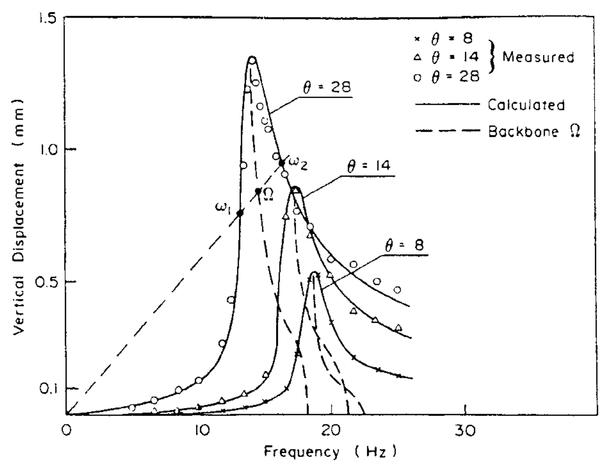

Figure 14 Vertical pile response measured and backcalculated for three levels of excitation intensity (Han & Novak, 1988)

amplitudes measured on a 3.38 m long test pile with increasing intensity of harmonic excitation. As the excitation forces grow, the resonance frequencies are markedly reduced, indicating strong nonlinearity. To the response curves, backbone curves, Ω, can be constructed and from them the pile restoring force-displacement relationships are established (Fig. 15). It appears that each response curve has its own backbone curve and corresponding stiffness

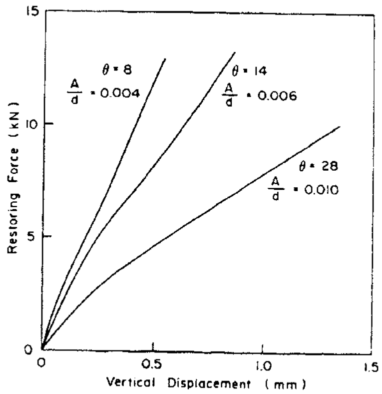

Figure 15 Pile restoring forces vs displacement corresponding to response curves from Fig. 14 (Han & Novak, 1988)

characteristic. The individual stiffness characteristics feature strong overall softening with excitation intensity (θ) but relatively modest nonlinearity.

The nonlinear pile stiffness characteristics were investigated for both horizontal and vertical dynamic response by Angelides and Roesset (1980) using toroidal finite elements in the region surrounding the soil and the consistent boundary matrix. Even neglecting slippage and gapping, they demonstrate the dramatic reduction in pile horizontal stiffness and equivalent damping with increasing pile force (Fig. 16). The p-y curves, also used for comparison, give lower stiffness because they account for gapping and a high number of load cycles, N, while only 10 load cycles were applied by Angelides and Roesset. The effect of a stable gap on soil resistance to pile steady-state vibration is schematically depicted in Fig. 17. The reduction of the equivalent linear stiffness and the necking of the loop are evident. Progressive degradation occurs under incrementally increasing loads when the hysteresis loops exhibit different shapes for sands and clays. This is exemplified by Fig. 18 showing the force-displacement relationship obtained by Kishida et al. (1985) on their model piles exposed to horizontal loads. In clay, the gap indicated in Fig. 17 may expand with each cycle giving rise to the characteristic elongated loops with reduced radiation damping.

If a generally nonlinear and particularly transient response rather than the steady-state response is to be investigated, time-domain analysis is called for. The lumped mass models, such as the one in Fig. 11, are readily amenable to such analysis. Another type of time-domain analysis, extending the dynamic Winkler model to allow for nonlinearities, was formulated by Nogami and Konagai (1986), Nogami, Konagai and Otani (1988), and Mitwally and Novak (1988). Under axial vibration, much of the nonlinearity is due to slip and friction. One model allowing for slip at the pile surface, nonlinearity near the pile and infinity of the outer zone is depicted in Fig. 19. One of the advantages of this model is that its properties are specified

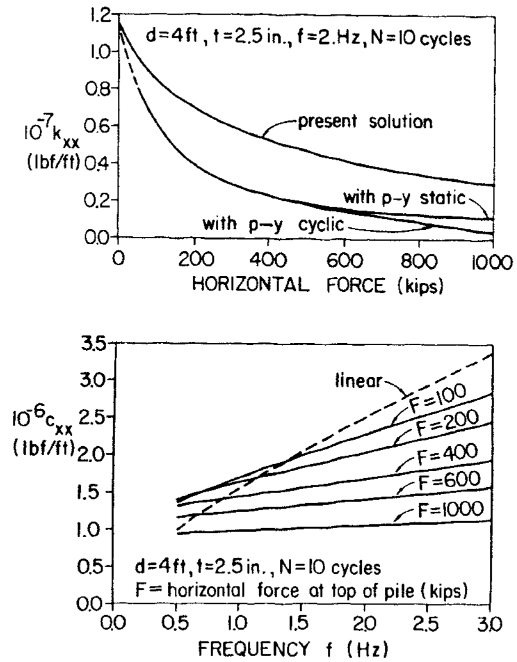

Figure 16 Variations of pile horizontal stiffness, k_{xx}, with force and pile equivalent damping, c_{xx}, with force and frequency due to soil nonlinearity (Angelides & Roesset, 1980)

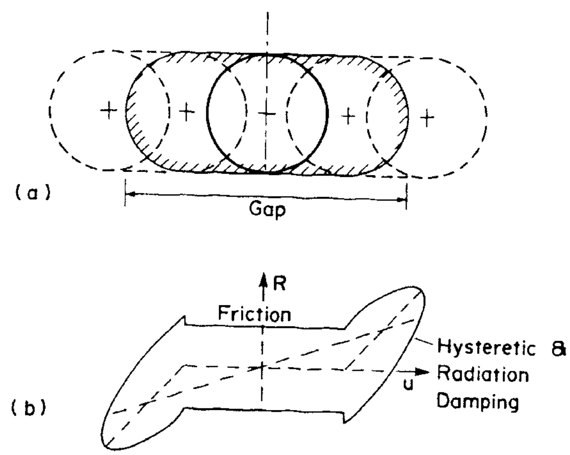

Figure 17 Schematic of (a) pile under steady-state vibration in stable gap, and (b) corresponding soil reaction, R, vs pile displacement, u, for stable cycle

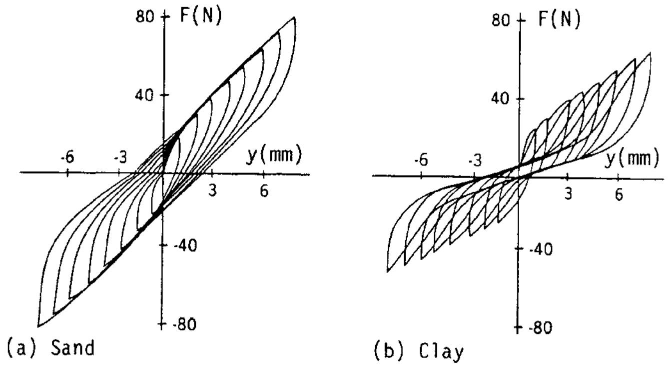

Figure 18 Force-displacement relationship for incrementally increasing horizontal load (Kishida et al., 1985)

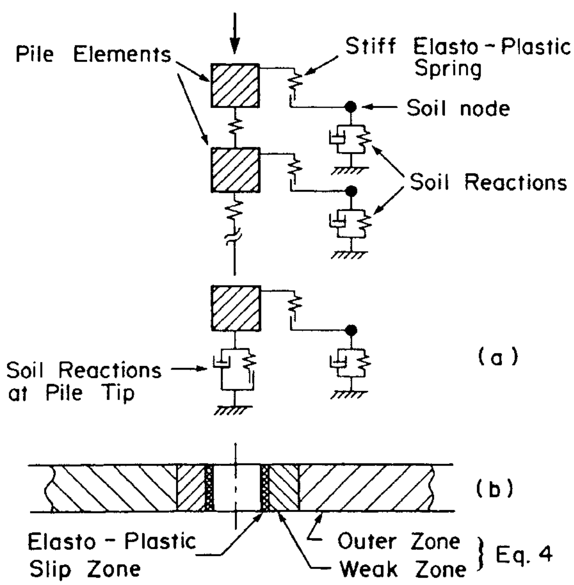

Figure 19 Pile model for vertical vibration allowing for slip, nonlinearity and infinity of the outer zone (Mitwally & Novak, 1988)

in terms of the standard geotechnical parameters.

PILE GROUPS

Piles are usually used in groups and if they are not very widely spaced they interact with each other generating phenomena known as pile-soil-pile interaction or group effects. These effects have attracted much interest in recent years. A number of papers on the subject have appeared, a few exhaustive Ph.D. dissertations were written (e.g. Kaynia, 1982a; Ostadan, 1983, Mamoon, 1990, Hassini, 1990) and many contributions have been made to the world conferences on earthquake engineering and are being presented to this conference.

Linear Behavior of Pile Groups

Under static loads, pile interaction increases group settlement, redistributes the loads on individual piles and reduces bearing capacity, unless this reduction is counteracted by densification of the soil within the group due to pile driving. The first suggestion of this kind of effects probably can be attributed to Sooysmith (1896). The investigation of static group effects was put on a rational basis, relying on continuum mechanics, by Poulos (1968, 1971, 1979) and Butterfield and Banerjee (1971). Extensive data on static group effects are available in Poulos and Davis (1980), Butterfield and Douglas (1981), El Sharnouby and Novak (1985, 1986, 1990) and elsewhere. The static data are useful even to those interested in dynamics because at low frequencies, and particularly below the fundamental frequency of a stratum (Fig. 2), the dynamic stiffness is usually quite close to the static stiffness.

Dynamic investigations of pile groups are more recent. The techniques employed are extensions of the approaches used for single piles and most of them are limited to linear interaction with no allowance for gapping. The methods rely on the availability of Green's functions with which the load transfer from the pile surface to soil can be calculated. These loading conditions, representing one of the basic differences between various approaches, range from point loads to line loads, ring loads, disk loads and finally to cylindrical (barrel) loads; for the pile base, disk loads are the rule. Applying this loading to individual segments into which the pile is discretized, the soil dynamic displacement field is established, yielding the soil dynamic flexibility matrix; inverting the latter, soil stiffness matrix is obtained. In this process, the presence of the pile cavities outside the loaded segment is usually ignored, which implies that wave scattering among the piles is not accounted for, and the soil displacements are calculated either for the pile axes or as averages of the circumferential values. A typical model for this analysis is shown in Fig. 20. Then the soil stiffness matrix is combined with the pile structural stiffness and the soil-pile system can be analyzed for any type of excitation. Different authors proposed various refinements or simplifications to this procedure.

The first theoretical analysis of pile-soil-pile interaction was conducted by Wolf and von Arx (1978) who employed an axisymmetric finite element formulation to establish the dynamic displacement field due to ring loads. Waas and Hartmann (1981, 1984) formulated an efficient semi-analytical method which uses ring loads and is well suited for layered media, properly accounting for the far field; the layers ought to be thin. Kaynia (1982a,b, 1988) and Kaynia and Kausel (1982, 1990) further improved the accuracy by combining the cylindrical loads, actually a boundary element formulation, with the consistent stiffness matrix of layered media to account for the far field. A very similar approach is employed in the paper to this conference by Kobori et al. (1991) who use the cylindrical

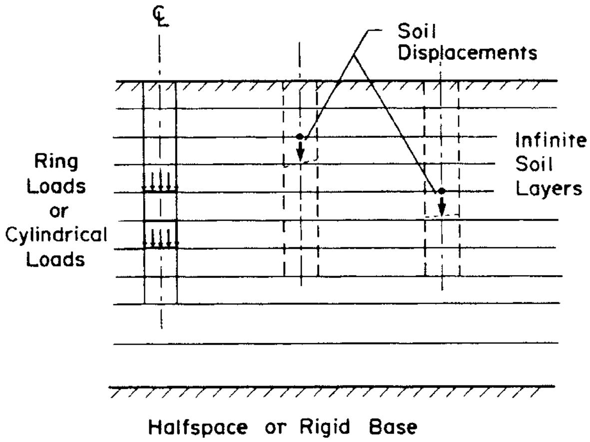

Figure 20 Typical model for pile group analysis

loads for the pile and disk loads for the base as depicted in Fig. 21. Also presented to this conference is a paper by Baba (1991) who formulates a three-dimensional analysis of endbearing piles.

The thin layer method was used by Tajimi and Shimomura (1976), Shimizu et al. (1977), Masuda et al. (1986) and a few others. Boundary element solutions, employing Green's functions of generally layered media, were formulated by Banerjee and Sen (1987), Banerjee et al. (1987), Mamoon et al. (1988, 1990, 1990a, 1990b) and Mamoon (1990), who examined a number of cases including pile batter and pile cap interaction. Simpler solutions based on the dynamic Winkler medium were developed by Nogami (1980, 1985) and Sheta and Novak (1982). The advantages of the latter approach are that it makes it possible to include the weak zone (Sheta and Novak, 1982) nonlinearity (Otani et al., 1991).

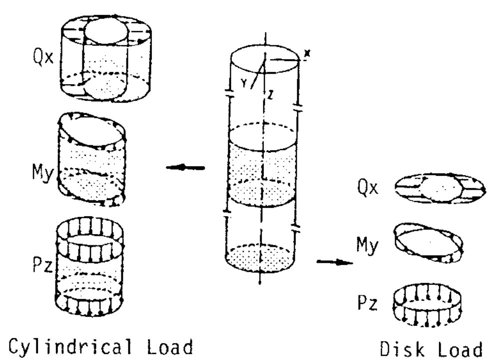

Figure 21 Cylindrical loads for pile surface and disk loads for base as used by Kobori et al. (1991)

Basic features of dynamic group effects

A few main features of the dynamic group effects emerge from the theoretical solutions: both stiffness and damping are strongly frequency dependent, can be either reduced or increased due to pile-soil-pile interaction, may exhibit very sharp peaks and are affected even for very large pile spacings. Some of these features can be seen in the example of a 4x4 group whose normalized dynamic stiffness is displayed for different spacings in Fig. 22. The normalization is done using the product (number of piles x single pile stiffness), and yields a ratio

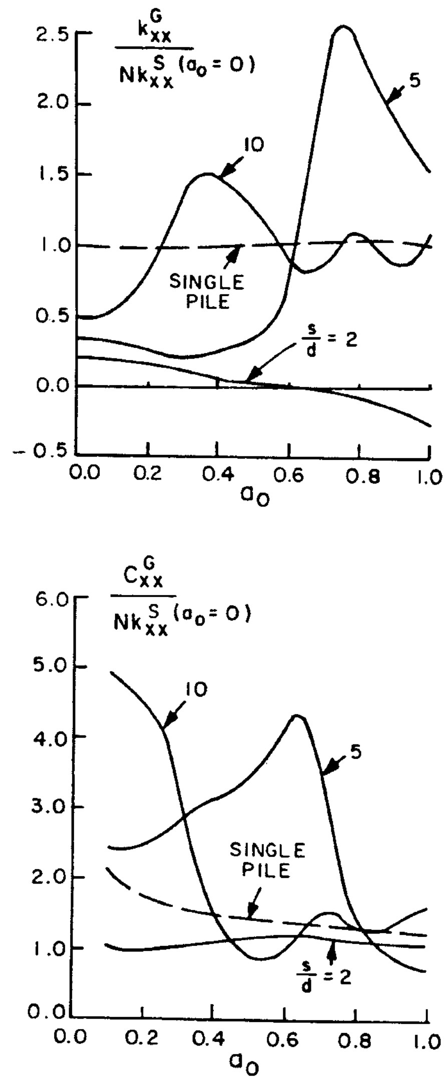

Figure 22 Normalized dynamic stiffness and damping of 4x4 pile group for different spacing ratios, s/d (Kaynia & Kausel, 1982; homogeneous halfspace, $a_o = \omega d/V_s$, $L/d = 15$, $E_p/E_s = 1000$, $\rho_s/\rho_p = 0.7$)

expressing group efficiency. As can be seen in Fig. 22, the group properties and their variation in frequency depend strongly on the spacing ratio, s/d, with the peaks shifting according to this ratio. This is so because pile interaction depends on the ratio of the wave length to spacing. It has been questioned whether the sharp peaks in stiffness would not be suppressed due to soil nonlinearity and interface deficiencies discussed above. A group solution including the weak zone around the piles dulls the peaks, but does not eliminate them, as is depicted in Fig. 23. On the other hand, soil

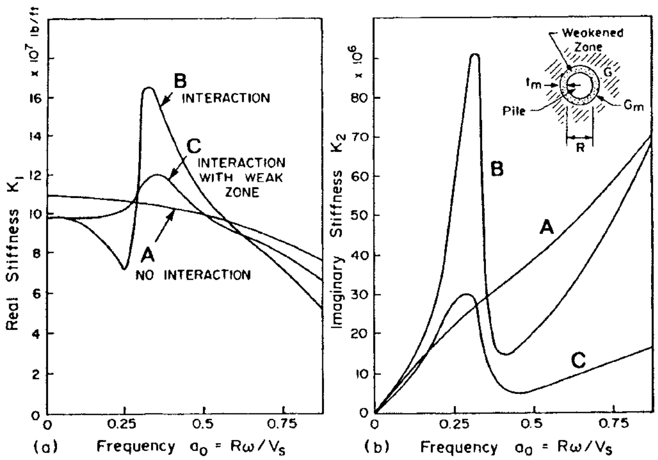

Figure 23 Effect of weak zone on vertical dynamic stiffness of 2x2 group of four concrete floating piles (s/d = 4, d = 2 ft = 0.61 m, parabolic soil profile; Sheta & Novak, 1982)

nonhomogeneity can make the peaks either more pronounced, as shown in Fig. 24, or duller as is argued by Gazetas and Makris (1991), depending on conditions such as frequency and spacing.

With the pile-soil-pile interaction theories being so complex, it is of importance to examine how the theories perform when compared with experiments. Figure 25 shows one such comparison based on a group of four closely spaced model piles tested in the field and evaluated using the plane strain theory for soil reactions with a weak zone. The responses evaluated ignoring interaction or assuming static interaction are completely inadequate. The dynamic interaction theory gives far better results. On a test group of 102 model piles 1.06 m in length similarly encouraging results were obtained (Novak and El Sharnouby, 1984). For six full scale piles very good results were also obtained but the weak zone and separation had to be included for a satisfactory match (Fig. 26). Successful experiments conducted on a group of 56 full scale piles were reported by Masuda et al. (1986). In their report to this conference, Kobori et al. (1991) also found the theory to be of sufficient applicability. Thus, it may be concluded that the linear theory works quite well as long as the experiments do not deviate too much from the theoretical assumptions, as might be expected. Often, a correction for separation, gapping and nonlinearity is needed, however, at least in the form of the weak zone and a pile free length. A few observations on nonlinear analysis will be made later herein.

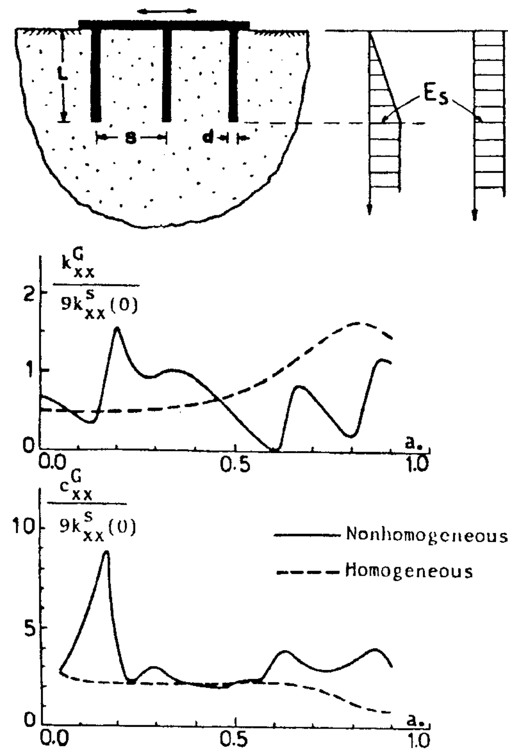

Figure 24 Horizontal dynamic stiffness of 3x3 pile group in homogeneous and nonhomogeneous soil (Kaynia, 1988; s/d = 5, L/d = 20, E_s/E_p = 0.01)

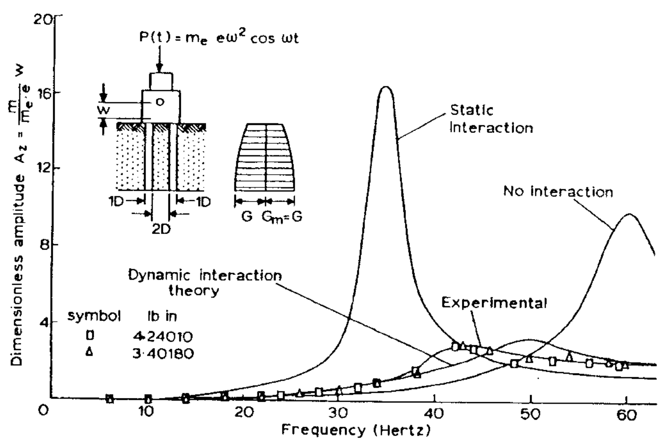

Figure 25 Vertical response of 2x2 group of closely spaced piles: theory vs. experiment (Sheta & Novak, 1982; L = 3.4 m, d = 60.3 mm)

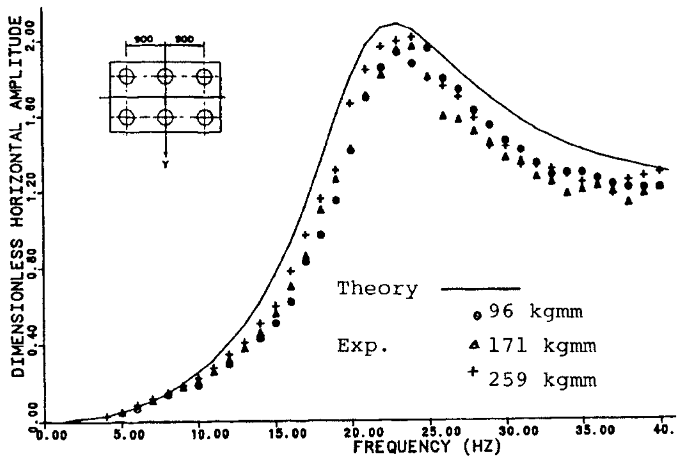

Figure 26 Horizontal theoretical and experimental response in Y-direction for group of six concrete piles 7.50 m long, 0.32 m in diameter (El Marsafawi et al., 1990)

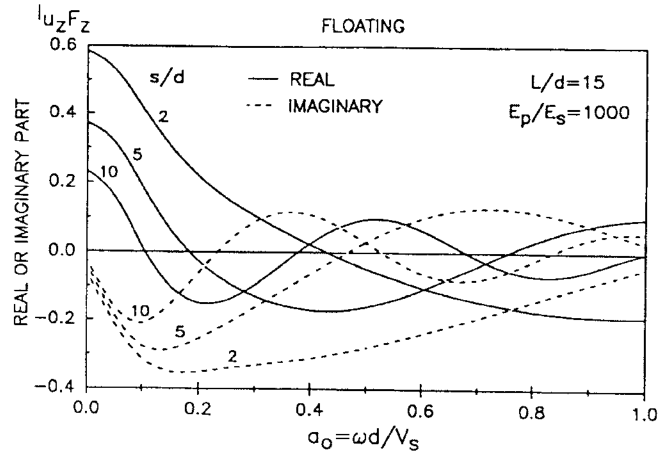

Figure 27 Vertical dynamic interaction factor for different spacings vs. dimensionless frequency (Kaynia & Kausel, 1982)

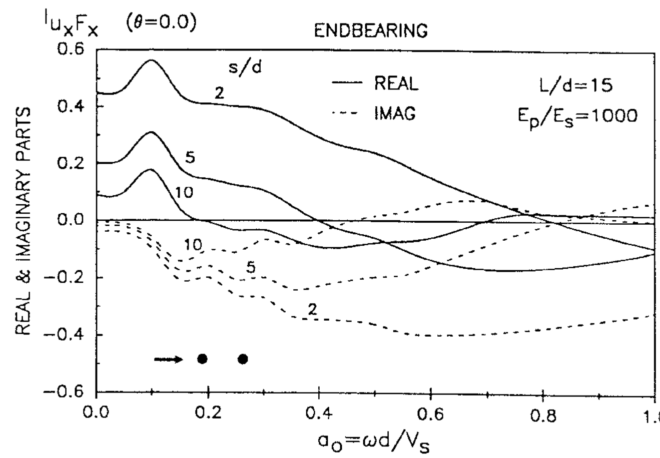

Figure 28 Horizontal dynamic interaction factors for two endbearing piles in line

Interaction Factors

The pile group analyses discussed above differ in accuracy and computing effort but for all of them the computing requirements are quite severe, particularly for larger groups. Therefore, Kaynia and Kausel (1982) formulated the concept of dynamic interaction factors being an extension of the widely used static interaction factor approach. In this approach, only two piles are considered at a time and the group properties are obtained by superposition.

Dynamic interaction factors are dimensionless, frequency dependent complex numbers, defined as

$$\alpha_{ij} = \frac{\text{Dynamic displacement of pile 2}}{\text{Static displacement of pile 1}} \tag{3}$$

in which the displacement of pile 2 is caused by a unit harmonic load on pile 1 and the static displacement of pile 1 is established for an isolated pile. The displacement is either translation or rotation. Examples of the real and imaginary parts of the interaction factors, calculated using the Kaynia and Kausel (1982) method, are plotted for homogeneous soil, using the authors' notations, in Figs. 27 to 29. The interaction factors are oscillatory in character, i.e. negative as well as positive. Negative values of the imaginary part indicate a possible increase in group damping characterized by group efficiency greater than unity. A complete set of interaction factors is available for floating piles, homogeneous soil and a limited selection of parameters in Kaynia and Kausel (1982) and for vertical vibration in linearly nonhomogeneous soil in Banerjee (1987).

The interaction factors, such as those shown in Figs. 27 to 29, are commonly displayed in terms of their real and imaginary parts. This is a usual form but it makes interpolation for intermediate spacings difficult, especially at higher frequencies. This difficulty can be circumvented if the interaction factors are expressed in terms of amplitude, $|\alpha|$, and phase, ϕ, i.e.

$$\begin{aligned} \alpha &= \alpha_1 + i\alpha_2 \\ &= |\alpha| e^{i\phi} \end{aligned} \tag{4}$$

As an example, the interaction factors from Fig. 27 are presented in this form in Fig. 30.

Correcting the available interaction factors for pile length, endbearing and other effects, a very efficient approximate procedure for group analysis is obtained. For example, the vertical or horizontal dynamic stiffness of a group with a rigid cap becomes

$$K^G = \bar{k} \sum_i \sum_r \varepsilon_{ir} \tag{5}$$

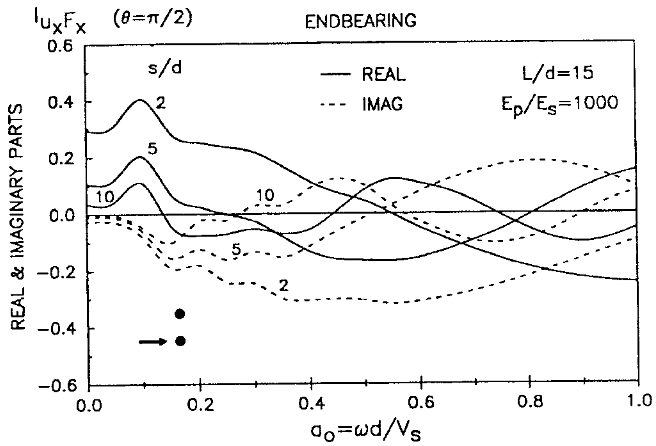

Figure 29 Horizontal dynamic interaction factors for two endbearing piles with 90° incidence angle

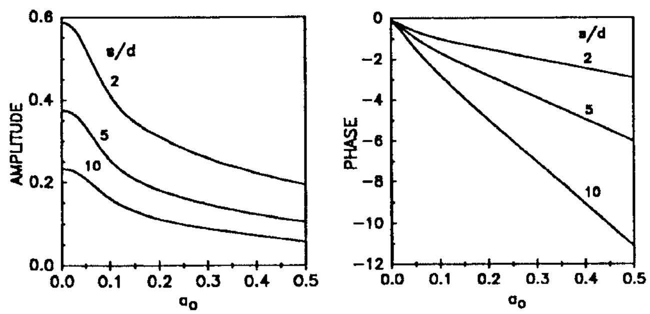

Figure 30 Vertical interaction factors from Fig. 25 in terms of amplitude and phase

in which $\bar{k}$ is static stiffness of a single pile and ϵ are the elements of the inverted matrix $[\alpha]$, listing all the complex interaction factors between any two piles in the group. (For all the vibration modes, the corresponding formulae can be found in Novak and Mitwally, 1990). When the loads on all the piles in a group are the same, as in a doubly symmetrical group of four piles, a simple formula for the group stiffness applies, i.e.

$$K^G = \frac{n\,\bar{k}}{f' + \sum_{j=2}^{n} \alpha_{ij}} \qquad (6)$$

in which $f' = \bar{k}/K$ is the ratio of the single pile static stiffness to its complex, dynamic stiffness, n is the number of piles and the interaction factors refer to one reference pile. Eq. 6 is often used as approximate even for more general pile configurations.

The interaction factor approach would be mathematically accurate if the interaction factors as well as the single pile properties were calculated with all piles present in the system, which is not normally done. Nevertheless, the results may be quite adequate for most applications. Kaynia and Kausel (1982) found the accuracy of the interaction factor approach to be quite sufficient for a homogeneous medium; for a nonhomogeneous medium, Kaynia (1988) observed the approach to be less accurate. Judging from static pile group behavior more significant errors, overestimating the interaction effects, may occur in the vertical response of endbearing piles (Fig. 31).

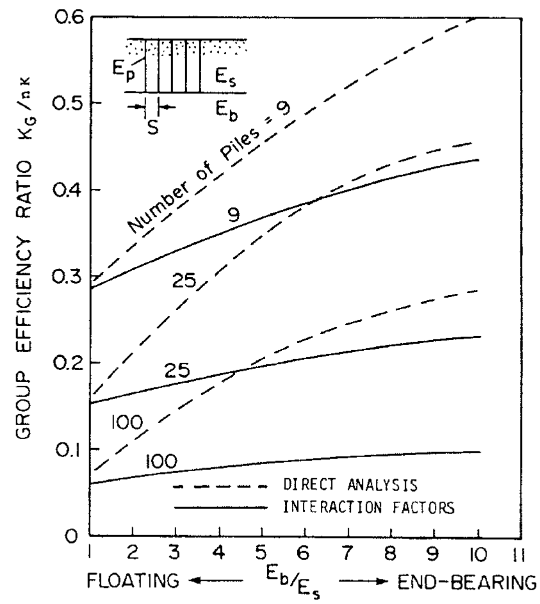

Figure 31 Comparison of vertical static group efficiency using direct analysis and interaction factors (El Sharnouby & Novak, 1985)

A remarkably simple approximate method for dynamic interaction factor evaluation was proposed by Dobry and Gazetas (1988) and extended for nonhomogeneous soils by Makris, Gazetas and Fan (1989) and Gazetas and Makris (1991). For homogeneous soils, these authors assume that the displacement field around a vibrating pile and thus also the displacement of the neighbouring pile (the interaction factor) is governed by the law of cylindrical wave propagation. Then, e.g., the vertical interaction factor is simply

$$\alpha_v \simeq \left(\frac{r_o}{s}\right)^{1/2} \exp\left(-\beta\omega\frac{s}{V_s}\right) \exp\left(-i\omega\frac{s}{V_s}\right) \qquad (7)$$

where β = soil hysteretic damping ratio. In their comparisons with the more rigorous solutions for floating piles, the authors obtained a very reasonable, although not quite perfect, agreement. For endbearing piles in a homogeneous stratum (Figs. 28 and 29), the frequency variations of the interaction factors are apparently too irregular to allow a simple description by a formula such as Eq. 7 and the same may be true for markedly stratified media.

Nonlinear Analysis of Pile Groups

Nonlinear dynamic analysis of pile groups is very difficult and this may be the main reason why it received much less attention than the linear analysis. Akiyoshi and Fuchida (1982) formulated an approximate solution for vertically vibrating endbearing piles considering imperfect adhesion between the pile and the soil modelled by a friction type interface. They found that slip occurs near the ground surface and proceeds to the bottom of the soil layer as the applied force increases. Nogami and Konagai (1987) developed a group analysis assuming also that in the vertical vibration, response nonlinearity stems mainly from slippage at the soil-pile interface; they represented the soil using the dynamic Winkler model. They found that this nonlinearity reduces the wave interference effects, making the stiffness less frequency dependent, and under transient loading produces residual skin friction and residual axial force in the pile. Then, Nogami et al. (1988) and Otani et al. (1991) extended the concept of the dynamic Winkler medium further to include horizontal response, slippage, gapping and inelastic soil behavior being able to generate a variety of degrading hysteresis loops.

The opinion is sometimes expressed that under large displacements most of the action occurs right around the pile and consequently, pile-soil-pile interaction is not very significant. Some insight into this can be obtained from static experiments. Figures 32 and 33 show the results of field tests conducted on free-headed test piles being steel pipes 0.1016 m in outer diameter and 3.05 m in length. The soil was stratified, mainly silty sand changing to gravel. Figure 32 shows two curves, one depicting the nonlinear response of the loaded pile and the other showing the interaction factor (normalized deflection), α, vs deflection. The interaction

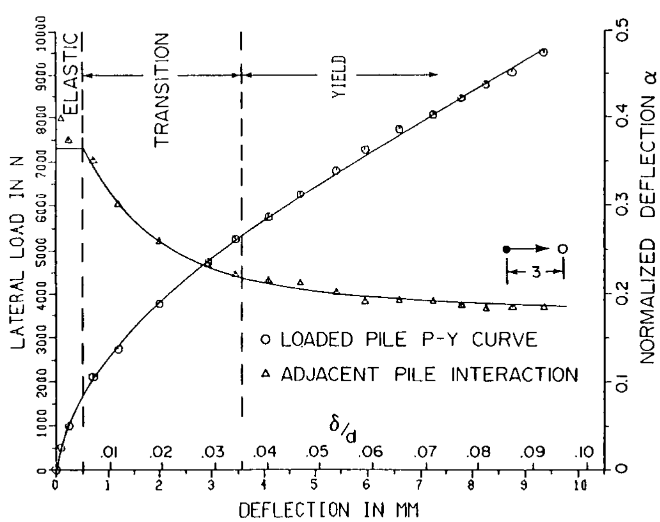

Figure 32 Lateral load vs deflection and static interaction factor α vs deflection (Janes & Novak, 1989)

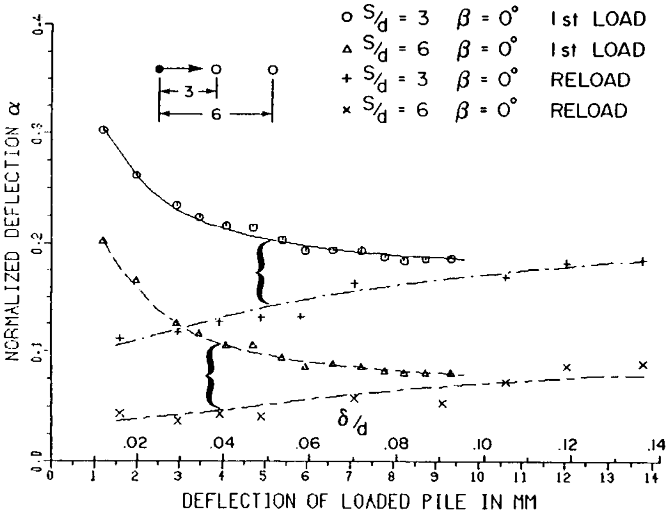

Figure 33 Horizontal static interaction factors for first loading and reloading (Janes & Novak, 1989)

factor diminishes with increasing deflection, dropping to about one half of the original value at the deflection of about 3.5 per cent of the diameter, and then levels off. This reduction varies with spacing and the angle of incidence. If the pile is unloaded and reloaded, the interaction factors for small displacements become much smaller than the original ones, apparently due to gaps generated by the preceding large displacements, and then asymptotically approach the values from the first loading. This behavior obviously makes the analysis of irregular transient response very difficult. Much more research in this area is needed. The preliminary conclusion is that under large displacements pile-soil-pile interaction is reduced but not eliminated.

OTHER FACTORS AFFECTING PILE BEHAVIOR

There are a few other factors that affect pile response, among them pile batter, soil-pile-cap interaction and soil liquefaction. These are briefly discussed in this section.

Pile Batter

Pile batter is often used to increase the horizontal stiffness of the group. For machine foundations and other structures this is sometimes useful. However, under earthquake loading, pile batter may not always be beneficial because it restricts the pile's ability to sway and yield, resulting in greater seismic forces and possible damage to the piles and the cap. Little information is available on the dynamic effects of the batter. As a very approximate practical approach, the pile can be analyzed first as_if it were vertical and the stiffness matrix [K] obtained in this way taken as valid for the inclined element coordinates; then, this matrix can be transformed into global coordinates, being horizontal and vertical, to give the battered pile stiffness matrix in these coordinates, [K]. More details on this are given by Novak (1980). For static conditions, Poulos (1980) employs a similar technique. He recommends the evaluation of interaction between two battered piles such as that of two vertical piles whose distance is equal to the separation measured on the inclined piles at L/3 from the top.

One of the few dynamic solutions of pile groups with batter was produced by Mamoon (1990) using an approximate analytical formulation, denoted as Method I. This procedure involves the construction of an integral representation for the soil domain modelled as an elastic halfspace. An example of Mamoon's results is shown in Fig. 34

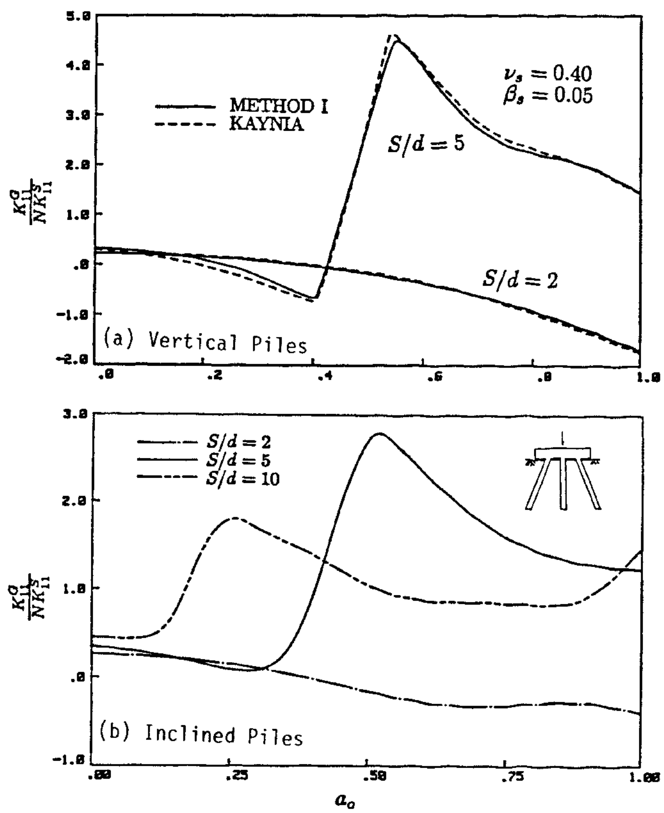

Figure 34 Normalized real part of vertical impedance of 3x3 pile group for (a) - vertical piles and (b) - piles with 15° incline (L/d = 15, E_p/E_s = 1000, ρ_s/ρ_p = 0.7) (Mamoon, 1990)

comparing the normalized vertical stiffness (real part) of a 3x3 group of vertical piles with that of a similar group featuring piles with a 15° batter in one plane. (Notice the vertical scale is not the same for both cases.) Kaynia's solution of the vertical group is displayed for comparison. The normalization is by the static stiffness of a single vertical pile multiplied by n. The comparison of cases (a) and (b) suggests that for the separation s/d = 5 and higher frequencies, the inclination of the piles causes a significant reduction in the real part of the impedance. For the peak, this reduction is about 43 per cent. Also, a slight shift in the peak can be noticed. The batter effect on the imaginary parts is similar but at frequencies higher than 0.6 the imaginary parts of the impedances are increased. For the horizontal response, the data available are not sufficient to make a general conclusion.

<u>Soil-Pile-Cap Interaction</u>

In most situations piles have caps and soil-pile-cap interaction may occur. The cap influence depends not only on the size and embedment of the cap but also on the quality of its contact with the soil. Considering the behavior of actual soils under static and dynamic loading, it may be speculated that this contact will be well maintained in stiff clays and dense sands; but in loose to moderately dense sands the cap base may separate from the soil and in soft clays the contact in the cap base as well as along the cap sides can be lost; finally, the separation of the base is more likely to occur for endbearing piles.

The few dynamic analyses that have been reported invariably presume full contact and perfect elasticity and thus their results should be applied with some allowance for the actual soil behavior. Banerjee and Sen (1987) observed a rather small effect of the cap on the vertical impedances of single piles and groups of two and four floating piles respectively. This might be a valid conclusion for the rather stiff piles they analyzed (E_p/E_s = 10000). For more flexible piles the cap may cause a more significant increase in pile impedances as can be deduced from static analysis. Figure 35 illustrates this point.

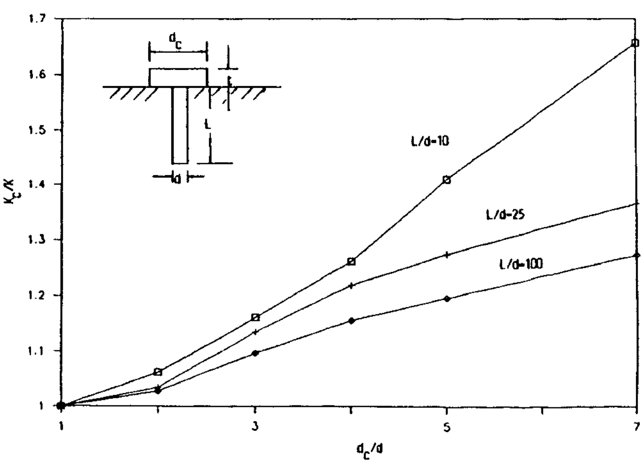

Figure 35 Ratio of vertical static stiffness of single pile with cap, K_c, to stiffness of pile without cap, K, for different cap diameters, d_c (E_p/E_s = 1000, ν = 0.5; Liu & Novak, 1990)

An extensive theoretical study of the dynamic cap effects was conducted by Mamoon (1990). He included cap inertia in his analysis but ignored the shear stresses in the mat base, even for the horizontal response. An example of Mamoon's results is shown in Fig. 36. The principle observation is that for some conditions, cap inertia can reduce or even eliminate the sharp peaks in the impedances, typical of pile groups without caps.

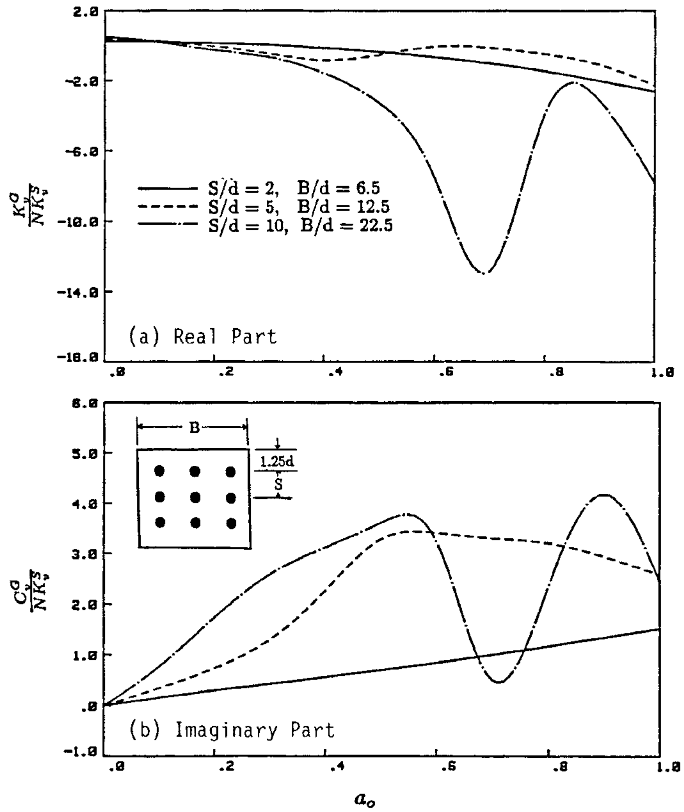

Figure 36 Normalized vertical impedances of 3x3 groups with caps for different spacings and cap sizes (Mamoon, 1990; L/d = 15; E_p/E_s = 1000, ρ_s/ρ_p = 0.7, ν_s = 0.4, β_s = 0.05, cap thickness = 3d)

An approximate practical approach to cap interaction is employed by Kobori et al. (1991) in their paper to this conference. To analyze the response of a group with an embedded cap, these authors superimpose three partial solutions to the entire problem as indicated in Fig. 37, and add side soil springs to account for the cap embedment. In the comparison of their analysis with experiments they get fair agreement.

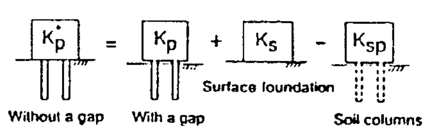

Figure 37 Approximate approach to cap interaction based on superposition of three partial solutions (Kobori et al., 1991)

Effect of Soil Liquefaction on Pile Behavior

Piles are often used in loose saturated sands and silts. If such deposits liquefy due to increased pore water pressure during earthquakes, the piles lose much of their lateral and vertical support which can result in a substantial increase in bending moments, loss of stability and failure. Damage of this type occurred in the Niigata and Alaska earthquakes of 1964 and elsewhere.

Relatively few studies were devoted to this important subject, e.g. Finn and Martin (1980), Matlock et al. (1981) and Yoshikawa and Arano (1988). To this conference Nomura et al. (1991) present their theoretical and experimental study of pile behavior during liquefaction. Their theory employs a one-dimensional effective stress analysis and Ramberg-Osgood's stress-strain relationship for soil. For the piles, a lumped mass model similar to that of Penzien et al. (1964) is used. The experiments were conducted in a 4.0 m long container on a large shaking table. The authors achieved excellent agreement between the theory and experiments with regard to ground motions, pore water pressure and pile response. The differences between the behavior of flexible piles and rigid piles were demonstrated and the one order of magnitude increase in both ground and pile motions due to liquefaction was documented.

SOIL-PILE-STRUCTURE INTERACTION

Once the properties of the pile foundation are established, they can be incorporated into the examination of pile-structure interaction just as with other types of foundations. A number of studies have been devoted to this subject. As there is a difference between direct excitation of the structure by external loads and excitation by seismic motions of the ground, these two cases will be discussed separately.

Pile-Structure Interaction Under External Loads

Typical examples of direct external loads are unbalanced forces acting on machine foundations, wind forces on buildings and wave forces on offshore towers. In such cases, the pile foundation impedances can be superimposed on the structural system matrices to give the governing equations of the pile-structure system in the standard form, i.e.

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{P(t)\} \qquad (8)$$

in which [m], [c] and [k] are the mass, damping and stiffness matrices incorporating the structure and foundation properties and, in some cases, other factors such as hydrodynamic effects, aerodynamic damping etc.; {u} and {P(t)} are the displacement vector and loading vector respectively. Two examples of structural response to external loading are given here, both with the aim of illustrating the effects of pile-soil-pile interaction.

Figure 38 shows the horizontal and rocking components of the response of a compressor foundation to harmonic unbalanced forces. The foundation is a concrete block 4.88 x 3.05 m in plan supported by eight endbearing wood piles

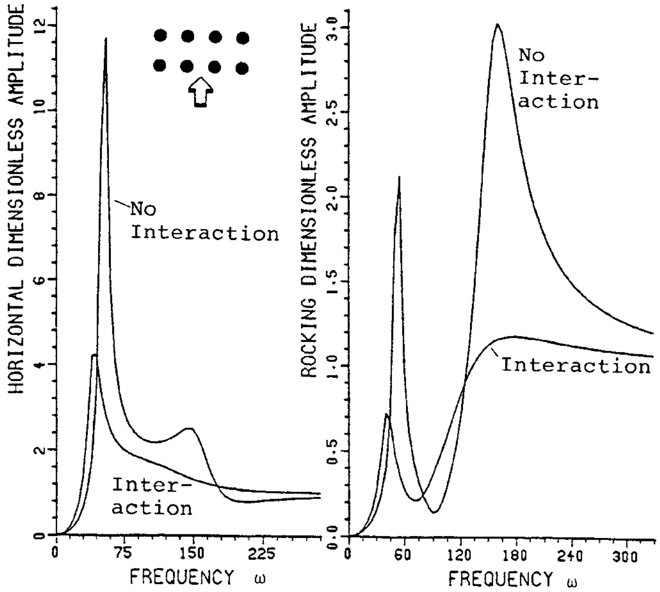

Figure 38 Effect of dynamic pile-soil-pile interaction on harmonic response of machine foundation

with a minimum spacing ratio s/d = 5.4. As can be seen from Fig. 38, dynamic pile-soil-pile interaction reduces the resonant amplitudes quite substantially but shifts the resonance frequencies only slightly.

The second example involves the response of a pile supported offshore tower to random wave forces. The tower is a steel template structure, 122 m high, supported by eight steel piles. (For details see Novak and Mitwally, 1990). The response of the tower to wave forces was analyzed in terms of random vibration twice, i.e. considering pile-soil-pile interaction and neglecting it. The power spectra of the tower response are shown in Fig. 39. The waves are wind driven. For a medium wind velocity of 22 m/s, the response spectrum features two peaks: one is centred around the fundamental frequency of the tower, ω_1, while the other coincides with the peak of the wave spectrum. The first one is dramatically reduced due to pile-soil-pile interaction because it is of resonant type and as such is sensitive to the increase in damping this interaction causes. The second peak occurs well below the fundamental tower frequency, is quasistatic and indicates response amplification due to increased flexibility. At the higher wind velocity of 30 m/s, the dominant frequency of the wave spectrum is very low, most of the response is quasistatic and is increased due to increased pile foundation flexibility (reduced stiffness). In addition to this stiffness reduction, gapping was observed to temporarily reduce tower natural frequencies during heavy storms.

Pile-Structure Interaction Under Seismic Loading

The evaluation of soil-pile-structure interaction is needed in order to establish the forces

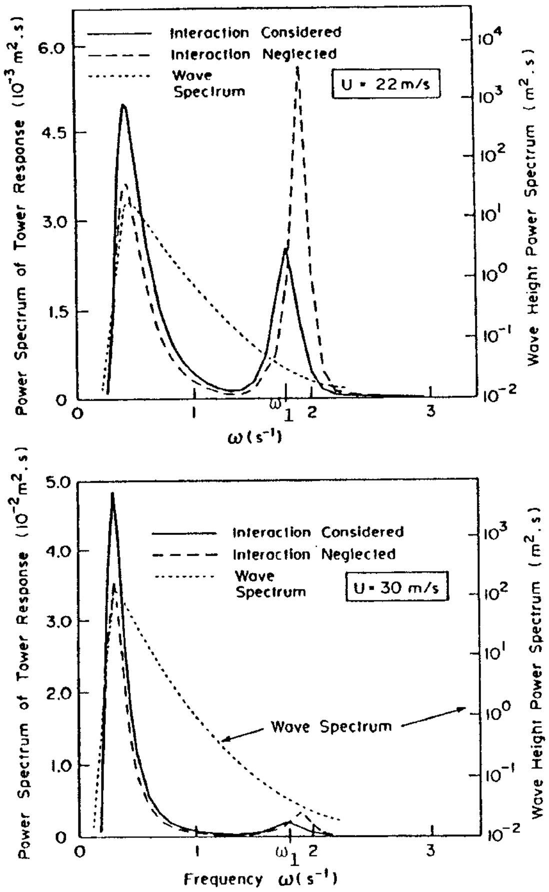

Figure 39 Effect of pile-soil-pile interaction on power spectra of steel offshore tower response to wave forces for two wind velocities (Novak & Mitwally, 1990)

expected to act on the structure and the piles in a seismic event. Such studies can be done experimentally or theoretically.

The experimental investigations are most often conducted on models using shaking table tests, less often in a centrifuge. The tests require careful scaling and special design of the test bin boundaries which should prevent wave reflections (the box effect). Shaking table tests of pile supported structures were reported by Mizuno et al. (1984), Nomura et al. (1991) and a few others; pile scaling was examined by Kana et al. (1986) and the modeling of free-field conditions in centrifuge tests was investigated by Cheney et al. (1990). Earthquake observations on a large scale model featuring 7.5 m long piles were made by Kobori et al. (1991).

For design purposes, the theoretical analysis of pile-structure interaction is more practical and is conducted much more often. Adequate for routine designs is a simple procedure based on substructuring and the following assumptions: the input ground motion is given for the level of pile heads and is not affected by the presence of the piles and their cap; soil-pile interaction analysis is conducted separately to yield the pile foundation impedances; and, the seismic response is obtained from Eq. 8 using standard analysis, even response spectra. For shear buildings all the matrices in Eq. 8 may be rearranged to take on the form that is common to shallow foundations (see, e.g., Novak and El Hifnawy, 1984). This type of analysis, known as inertial interaction analysis, usually indicates that the pile foundation flexibility and dissipative properties result in the reduction of the seismic forces as well as the base shear and an increase in the relative building response, just as in the case of shallow foundations (Novak and El Hifnawy, 1984).

The assumption of the input ground motion not being affected by the presence of the piles is based on the ideas that the dominant seismic wave lengths are much larger than the pile diameter, and given the bending flexibility of slender piles, the piles will follow the horizontal motion of the ground. A more comprehensive examination of these assumptions involves consideration of the wave scattering effect, known as kinematic interaction. (Unfortunately, there is not a unique definition of this term.) A few researchers examined this phenomenon. Gazetas (1984) conducted an extensive parametric study of the response of single endbearing piles exposed to harmonic shear waves propagating upward from the bedrock. He defined the kinematic interaction factor as

$$I_u = \frac{u_p}{u_o} \text{ and } I_\phi = \frac{\phi_p r_o}{u_o} \tag{10}$$

in which u_p, u_o are the absolute values of the horizontal displacements, relative to the bedrock, of the embedded pile head and the ground surface motion in the absence of the piles, respectively, and u_ϕ is the pile head rotation absolute value. The magnitude of I_u depends on the soil profile, the stiffness ratio E_p/E_s, the slenderness ratio L/d and the frequency ratio f/f_1 where f = wave frequency and f_1 = fundamental horizontal frequency of the soil layer being for a parabolic soil profile equal to 0.56 V_s/L. When there is no kinematic interaction $I_u = 1$. Synthesizing his numerical results, Gazetas found it possible to express the kinematic interaction factors for each soil profile in terms of a dimensionless frequency parameter. For the parabolic soil profile this parameter becomes

$$F_B = \frac{f}{f_1} \left(\frac{E_p}{E_s}\right)^{0.16} \left(\frac{d}{L}\right)^{0.35} \tag{11}$$

In terms of this parameter the kinematic interaction factor for translation assumes the form plotted in Fig. 40. As can be seen, for small f/f_1, E_p/E_s and d/L, the kinematic interaction factor is close to unity; for large values of these ratios it drops to about 0.5. In the studies conducted by other authors this drop can be even more pronounced. This suggests that the error resulting from the omission of kinematic interaction is either negligible or is on the conservative side. Only for the homogeneous soil profile, slight amplification of I_u may occur at low frequencies. The effect of the angle of incidence was examined by Mamoon and Banerjee (1990a), Mamoon and Ahmad (1990) and Ahmad (1991).

For pile groups, kinematic interaction can be more significant. Waas and Hartmann (1984) examined a single pile and a large group of 356 piles and concluded that while a single pile follows the earthquake motion of the soil with little deviation, a large group of stiff piles in soft soil shows a response significantly different from the free-field motion. Significant kinematic interaction effects were also observed for a similar pile group by Wolf and von Arx (1982) who considered horizontally traveling waves. Thus, for important projects such as nuclear power plants, a complete analysis including kinematic interaction may be desirable. Such a complete response analysis of a pile-supported structure, in which the kinematic interaction is evaluated beforehand to give the ground motion for the inertial interaction calculation, is schematically indicated in Fig. 41 with M representing the mass of the structure and a_o input bedrock acceleration. Analysis of this type was conducted by Waas and Hartmann (1984), Hadjian et al. (1990), Kobori et al. (1991) and others.

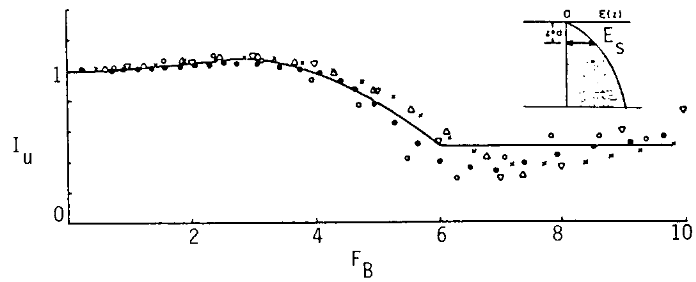

Figure 40 Kinematic interaction factor for parabolic soil profile vs dimensionless frequency parameter F_B; $E_s = E_s$ (z=d); (Gazetas, 1984)

The two step response analysis shown in Fig. 41 indicates that pile stresses also come from two sources, i.e. pile deflection due to ground motion and inertial interaction. One limitation of the accuracy of most kinematic interaction studies is that they assume soil linearity. It is well known that for strong earthquakes linear site response analysis can yield unrealistic displacements and stresses.

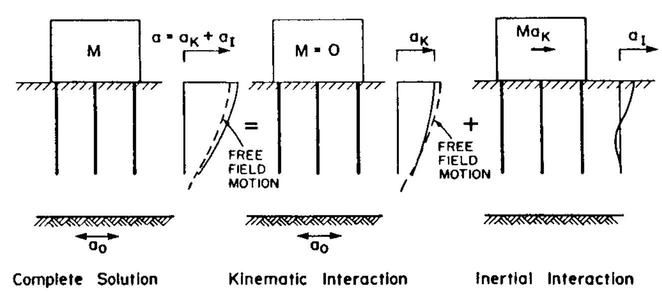

Figure 41 Schematic of seismic response analysis including kinematic interaction

One more complication may occur if the piles are not adequately connected to the cap or if this connection fails in a severe earthquake. Then the cap may uplift, as indicated in Fig. 42, modifying the seismic forces on the building and substantially increasing the forces on the peripheral piles that maintain the connection. These piles can become overloaded and may fail. Uplift of the tip of an endbearing pile, which was not socketed, from the bearing stratum may have similar but less severe results. More data on the uplift effects can be found in El Hifnawy and Novak (1986, 1987).

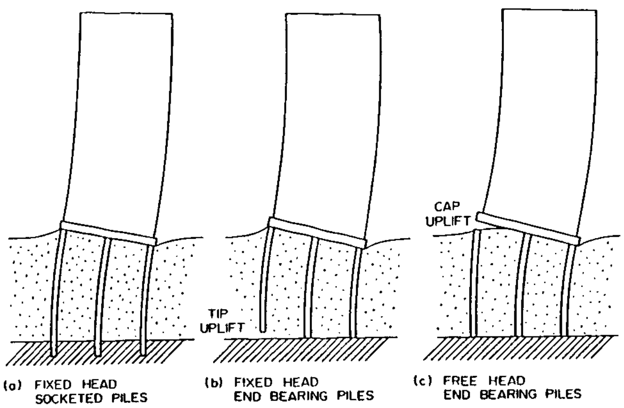

Figure 42 Uplift of pile cap and pile tip under seismic loading

CONCLUSIONS

Considerable progress has been made in the development of dynamic analyses of single piles and pile groups, experimental techniques for laboratory and field pile investigations and understanding of pile behavior. Further research is needed, particularly into soil-pile interface behavior, nonlinear pile-soil-pile interaction and the interaction between the piles and their caps, both surface and embedded.

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REFERENCES

Akiyoshi, T. and Fuchida, K. (1982). "An Approximate Solution of Vertical Vibration of End-Bearing Piles With Frictional Interface", Proc. JSCE, no. 324, August, 31-40 (in Japanese).

American Petroleum Institute (1986). "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms" (16th edition), Dallas, TX.

Angelides, D. and Roesset, J.M. (1980). "Nonlinear Dynamic Stiffness of Piles", MIT Res. Rep. R80-13, Dept. of Civil Eng., Cambridge, Massachusetts.

Aubry, D. and Chapel, F. (1981). "3-D Dynamic Analysis of Groups of Piles and Comparisons With Experiments", SMIRT, Paris, 9 p.

Banerjee, P.K. (1978). "Analysis of Axially and Laterally Loaded Pile Groups", Chapter 9 in Developments in Soil Mechanics, Ed. C.R. Scott, Appl. Sci. Pub., London, 317-346.

Banerjee, P.K. and Mamoon, S.M. (1990). "A Fundamental Solution Due to a Periodic Point Force in the Interior of an Elastic Half Space", Earthq. Eng. Struct. Dyn., vol. 19, no. 1, 91-105.

Banerjee, P.K. and Sen, R. (1987). "Dynamic Behaviour of Axially and Laterally Loaded Piles and Pile Groups", Chapter 3 in Dynamic Behavior of Foundations and Buried Structures (Developments in Soil Mech. Found. Eng., vol. 3), Ed. P.K. Banerjee and R. Butterfield, Elsevier App. Sc., London, 95-133.

Banerjee, P.K., Sen, R. and Davies, T.G. (1987). "Static and Dynamic Analyses of Axially and Laterally Loaded Piles and Pile Groups", Chapter 7 in Geotechnical Modeling and Applications (Dean Alexander Vesic Memorial Volume), Ed. S.M. Sayed, Gulf Pub. Co., Houston, p. 322.

Blaney, G.W., Kausel, E. and Roesset, J.M. (1976). "Dynamic Stiffness of Piles", Proc., 2nd Int. Conf. on Num. Methods in Geomech., Blacksburg, Virginia, 1001-1012.

Blaney, G.W., Muster, G.L. and O'Neill, M.W. (1987). "Vertical Vibration Test of a Full-Scale Pile Group", Geotech. Special Pub., ASCE, no. 11, 149-165.

Butterfield, R. and Banerjee, P.K. (1971). "The Elastic Analysis of Compressible Piles and Pile Groups", Geotechnique, vol. 21, 43-60.

Butterfield, R. and Douglas, R.A. (1981). "Flexibility Coefficients for the Design of Piles and Pile Groups", Construction Industry Res. and Inf. Assoc., 6 Storey's Gate, London, England, Technical Notes, 108.

Cheney, J.A., Brown, R.K., Dhat, N.R. and Hor, O.Y.Z. (1990). "Modeling Free-Field Conditions in Centrifuge Models", J. Geotech. Eng., vol. 116, no. 9, 1347-1367.

Crouse, C.B. and Cheang, L. (1987). "Dynamic Testing and Analysis of Pile-Group Foundations", Geotech. Special Pub., ASCE, 1987, No. 11, 79-98.

Davies, T.G., Sen, R. and Banerjee, P.K. (1985). "Dynamic Behavior of Pile Groups in Inhomogeneous Soil", J. of Geotech. Eng., Dec., vol. 111, no. 12, 1365-1379.

De Beer, E. et al. (Eds.) (1977). Proc. of Specialty Session 10, "The Effects of Horizontal Loads on Piles Due to Surcharge or Seismic Effects", IX ICSMFE, Tokyo, July 14, p. 200.

Dobry, R. and Gazetas, G. (1988). "Simple Method for Dynamic Stiffness and Damping of Floating Pile Groups", Geotechnique, vol. 38, no. 4, 557-574.

Dobry, R., Vincente, E., O'Rourke, M.J. and Roesset, J.M. (1982). "Horizontal Stiffness and Damping of Single Piles", J. Geotech. Eng. Div., ASCE, vol. 108, no. GT3, 439-459.

Dotson, K.W. and Veletsos, A.S. (1990). "Vertical and Torsional Impedances for Radially Inhomogeneous Viscoelastic Soil Layers", Soil Dyn. & Earthq. Eng., May, vol. 9, no. 3, 110-119.

El Hifnawy, L. and Novak, M. (1986). "Uplift in Seismic Response of Pile Supported Buildings", J. Earthq. Eng. & Struct. Dyn., vol. 14, August, 573-593.

El Hifnawy, L. and Novak, M. (1987). "Seismic Response of Buildings With Pile Uplift", Proc. 5th Canadian Conf. on Earthq. Eng., Ottawa, July, 181-190.

El-Marsafawi, H., Han, Y. and Novak, M. (1990). "Dynamic Experiments on Two Pile Groups", Res. Rep. GEOT-20-90, Dept. Civil Eng., U. Western Ontario, August.

El Sharnouby, B. and Novak, M. (1984). "Dynamic Experiments With Group of Piles", J. Geotech. Eng., ASCE, June, vol. 110, no. 6, 719-737.

El Sharnouby, B. and Novak, M. (1985). "Static and Low Frequency Response of Pile Groups", Canadian Geotech. J., vol. 22, no. 1, 79-94.

El Sharnouby, B. and Novak, M. (1986). "Flexibility Coefficients and Interaction Factors for Pile Group Analysis", Canadian Geotech. J., vol. 23, 441-450.

El Sharnouby, B. and Novak, M. (1990). "Stiffness Constants and Interaction Factors for Vertical Response of Pile Groups", Canadian Geotech. J., (in press).

Fellenius, B.H. (Ed.) (1988). Proc. Third Int. Conf. on Application of Stress-Wave Theory to Piles. Ottawa, Canada, p. 926.

Finn, W.D.L. and Martin, G.R. (1980). "Offshore Pile Foundations in Sand Under Earthquake Loading", Appl. Ocean Res., vol. 2, no. 2, 81-84.

Flores-Berrones and Whitman, R.V. (1982). "Seismic Response of End-Bearing Piles", J. of Geotech. Eng. Div., ASCE, vol. 108, no. 4, 554-569.

Gazetas, G. (1984). "Seismic Response of End-Bearing Single Piles", Soil Dyn. and Earthq. Eng., vol. 3, no. 2, 82-93.

Gazetas, G. and Dobry, R. (1984). "Horizontal Response of Piles in Layered Soils", J. Geotech. Eng. Div. ASCE, vol. 101, no. 1, 20-40.

Gazetas, G. and Makris, N. (1991). "Dynamic Pile-Soil-Pile Interaction, I: Analysis of Axial Vibration", J. Earthq. Eng. & Struct. Dyn., vol. 20, no. 2 (in press).

Gle, D.R. (1981). "The Dynamic Lateral Response of Deep Foundation", Ph.D. Thesis, Univ. of Michigan, Ann Arbor.

Hadjian, A.H., Falgren, R.B. and Lau, L. (1990). "Imperial County Services Building Revisited: A Reevaluation With Pile-Soil-Structure Interaction", Proc. 4th U.S. Nat. Conf. on Earthq. Eng., Palm Springs, Cal., vol. 3, 835-844.

Han, Y. and Novak. (1988). "Dynamic Behaviour of Single Piles Under Strong Harmonic Excitation", Canadian Geotech. J., vol. 25, no. 3, 523-534.

Hassini, S. (1990). "Static and Dynamic Behaviour of Pile Groups", Ph.D. Thesis, Civil Eng., Univ. of Michigan, Ann Arbor, Michigan.

Hassini, S. and Woods, R.D. (1989). "Dynamic Experiments With Model Pile Foundations", Proc. 12th ICSMFE, Rio de Janeiro, vol. 2, 1135-1138.

He, Y.A. (1990). "The Mixed Boundary Problem of Soil-Pile Interaction", Soil Dyn. & Earthq. Eng., vol. 9, no. 1, 20-24.

Hewitt, C.M. (1988). "Cyclic Response of Offshore Pile Groups", Ph.D. Thesis, School of Civil and Mining Eng., Univ. of Sydney, June, 254 p.

Hijikata, K., Miura, K., Miyamoto, Y. and Moroi, T. (1990). "Dynamic Characteristics of Pile-Groups Foundation (in Japanese)", Proc. of Japan Conf. of Struc. Div. in Archit., No. 408, 89-98.

Janes, M. and Novak, M. (1989). "Pile Group Response to Large Lateral Loads", Proc. 42nd Canadian Geotech. Conf., Winnipeg, Manitoba, Oct. 23-25, 216-222.

Kana, D.D., Boyce, L. and Blaney, G.W. (1986). "Development of a Scale Model for Dynamic Interaction of a Pile in Clay", J. Energy Resources Technology, vol. 108, 254-261.

Kausel, E., Roesset, J.M. and Waas, G. (1975). "Dynamic Analysis of Footings on Layered Media", J. Eng. Mech. Div., ASCE, vol. 101, no. EM5, 679-693.

Kaynia, A.M. (1982a). "Dynamic Stiffness and Seismic Response of Pile Groups", Ph.D. Thesis, MIT, Cambridge, MA.

Kaynia, A.M. (1982b). "Dynamic Stiffness and Seismic Response of Pile Groups", MIT Res. Rep. R82-03, Cambridge, MA.

Kaynia, A.M. (1988). "Characteristics of the Dynamic Response of Pile Groups in Homogeneous and Nonhomogeneous Media", Proc. 9th World Conf. on Earthq. Eng., Tokyo-Kyoto, Japan, Aug. 2-9, vol. III, 575-580.

Kaynia, A.M. and Kausel, E. (1982). "Dynamic Behavior of Pile Groups", 2nd Int. Conf. on Num. Methods in Offshore Piling, Austin, TX, 509-532.

Kaynia, A.M. and Kausel, E. (1990). "Dynamics of Piles and Pile Groups in Layered Soil Media", Int. J. Soil Dyn. & Earthq. Eng., in print.

Kishida, H., Suzuki, Y. and Nakai, S. (1985). "Behavior of a Pile Under Horizontal Cyclic Loading", Proc. XIIth ICSMFE, San Francisco, vol. 11, 1413-1416.

Kobori, T., Minai, R. and Baba, K. (1977). "Dynamic Behaviour of a Laterally Loaded Pile", Proc. of Specialty Session 10, 9th ICSMFE, Tokyo, July 14, 175-180.

Kobori, T., Minai, R. and Baba, K. (1981). "Dynamic Behavior of a Pile Under Earthquake Type Loading", Proc. Int. Conf. on Recent Advances in Geotech. Earthq. Eng. and Soil Dyn., U. of Missouri-Rolla, vol. 2, 795-800.

Krishnan, R., Gazetas, G. and Veliz, A. (1983). "Static and Dynamic Lateral Deflection of Piles in Non-Homogeneous Soil Stratum", Geotechnique, vol. 33, no. 3, 307-325.

Kuhlemeyer, R.L. (1976). "Static and Dynamic Laterally Loaded Piles", Res. Rep. No. CE76-9, Dept. of Civil Eng., U. of Calgary, Canada.

Kuhlemeyer, R.L. (1979a). "Vertical Vibration of Piles", J. Geotech. Eng. Div., ASCE, vol. 105, 273-287.

Kuhlemeyer, R.L. (1979b). "Static and Dynamic Laterally Loaded Floating Piles", J. Geotech. Eng. Div., ASCE, vol. 105, 289-304.

Lakshmanan, N. and Minai, R. (1981). "Dynamic Soil Reactions in Radially Non-Homogeneous Soil Media", Bul. Disaster Prevention Res. Inst., Kyoto University, Vol. 31, 79-114.

Liu, W. and Novak, M. (1990). "Pile-Soil Static Interaction Analysis by Finite and Infinite Elements", Res. Rep. GEOT-19-90, Dept. Civil Eng., Univ. West. Ontario, London, Canada.

Makris, N., Gazetas, G. and Fan, K. (1989). "Analytical Results for Pile-Soil-Pile Interaction in Vertical Harmonic Motion", ERCAD Conf., Berlin.

Mamoon, S.M. (1990). "Dynamic and Seismic Behavior of Deep Foundations", Ph.D. Thesis, Fac. of Grad. School, State Univ. of New York at Buffalo, N.Y.

Mamoon, S.M. and Ahmad, S. (1990). "Response of Single Piles to Obliquely Incident SH, SV and P Waves", J. of Geotech. Eng. Div., ASCE, vol. 116, no. 2, 186-204.

Mamoon, S.M. and Banerjee, P.K. (1990a). "Response of Piles and Pile Groups to Traveling SH-Waves", J. of Earthq. Eng. and Struct. Dyn., vol. 19, no. 4, 597-610.

Mamoon, S.M. and Banerjee, P.K. (1990b). "Dynamic Behavior of Piles and Pile Groups With Cap Interaction", J. of Geotech. Eng. Div., ASCE, to appear.

Mamoon, S.M., Banerjee, P.K. and Ahmad, S. (1988). "Seismic Response of Pile Foundations", Tech. Rep. NCEER-88-0034, Nat. Center for Earthq. Eng. Res., SUNY, Buffalo.

Mamoon, S.M., Kaynia, A.M. and Banerjee, P.K. (1990). "On Frequency Domain Dynamic Analysis of Piles and Pile Groups", J. Eng. Mech. Div., ASCE, vol. 116, no. 10, to appear.

Masuda, K., Saseki, F., Urao, K., Veno, K. and Miyamoto, Y. (1986). "Simulation Analysis of Forced Vibration Test of Actual Pile Foundation by Thin Layer Method", Proc. Annual Meeting of Architectural Inst. of Japan.

Matlock, H. and Foo, S.H.C. (1980). "Axial Analysis of Piles Using a Hysteretic Degrading Soil Model", Proc. Int. Symp. Numer. Methods Offshore Piling, Institute of Civil Engineers, London, 127-133.

Matlock, H., Foo, H.C. and Bryant, L.M. (1978). "Simulation of Lateral Pile Behaviour Under Earthquake Motion", proc. Am. Soc. Civ. Engrs. Specialty Conf. on Earthq. Eng. & Soil Dyn., Pasadena, CA, II, 600-619.

Matlock, H., Martin, G.R., Lam, I.P. and Tsai, C.F. (1981). "Soil-Pile Interaction in Liquefiable Cohesionless Soils During Earthquake Loading", Proc. 1st Int. Conf. Recent Adv. in Geot. Earthq. Eng. & Soil Dyn., St. Louis, Missouri.

Mitwally, H. and Novak, M. (1988). "Pile Driving Analysis Using Shaft Models and FEM", Proc. 3rd Int. Conf. on Appl. of Stress-Wave Theory to Piles, Ottawa, Canada, 455-466.

Mitwally, H. and Novak, M. (1987). "Response of Offshore Towers With Pile Interaction", J. of Eng. Mech., July, vol. 113, no. 7, 1065-1084.

Mizuhata, K. and Kusakabe, K. (1984). "Comparison of Experimental and Analytical Results of Vibration of a Full Scale Pile", Proc. 8th WCEE, San Francisco, vol. III, 633-640.

Mizuno, H. (1987). "Pile Damage During Earthquakes in Japan (1923-1983)", Dynamic Response of Pile Foundations, ASCE, Geotech. Special Publ. No. 11 (Edited by T. Nogami), 53-78.

Mizuno, H., Iiba, M. and Kitagawa, Y. (1984). "Shaking Table Testing of Seismic Building-Pile-Two-Layered-Soil Interaction", 8WCEE, San Francisco, vol. III, 649-656.

Morrison, C.S. and Reese, L.C. (1988). "A Lateral-Load Test of a Full-Scale Pile Group in Sand", Geot. Eng. Rep. GR86-1, Geotech. Eng. Center, Univ. of Texas at Austin, TX.

Nogami, T. (1980). "Dynamic Stiffness and Damping of Pile Groups in Inhomogeneous Soil", Proc. Session Dyn. Response Pile Found.: Anal. Aspects ASCE, 31-52.

Nogami, T. (1985). "Flexural Responses of Grouped Piles Under Dynamic Loading", Earthq. Eng. and Struct. Dyn., vol. 13, 321-336.

Nogami, T. (Ed.) (1987). "Dynamic Response of Pile Foundations - Experiment, Analysis and Observation", ASCE, Geot. Spec. Publication, No. 11, 186 p.

Nogami, T. and Konagai, K. (1987). "Dynamic response of vertically loaded nonlinear pile foundations", J. Geotech. Eng., vol. 113, no. 2, 147-160.

Nogami, T. and Konagai, K. (1988). "Time Domain Flexural Response of Dynamically Loaded Single Piles", J. Eng. Mech. Div., ASCE, vol. 114, no. 9, 1512-1525.

Nogami, T. and Novak, M. (1976). "Soil-Pile Interaction in Vertical Vibration", J. Earthq. Eng. & Struct. Dyn., vol. 4, 277-293.

Nogami, T., Konagai, K. and Otani, J. (1988). "Non Linear Pile Foundation Model for Time-Domain Dynamic Response Analysis", 9th WCEE, Tokyo, vol. III, 593-598.

Novak, M. (1974). "Dynamic Stiffness and Damping of Piles", Canadian Geotech. J., vol. 11, No. 4, 574-598.

Novak, M. (1977). "Vertical Vibration of Floating Piles", J. Eng. Mech. Div. ASCE, Feb., vol. 103, no. EM1, 153-168.

Novak, M. (1980). "Soil-Pile Interaction Under Dynamic Loads", Proc. Num. Methods in Offshore Piling, ICE, London, 59-68.

Novak, M. and Aboul-Ella, F. (1978a). "Impedance Functions of Piles in Layered Media", J. Eng. Mech. Div. ASCE, June, vol. 104, no. EM3, Proc. Paper 13847, 643-661.

Novak, M. and Aboul-Ella, F. (1978b). "Stiffness and Damping of Piles in Layered Media", Proc. Earthq. Eng. & Soil Dyn., ASCE Specialty Conf., Pasadena, CA, June 19-21, 704-719.

Novak, M. and El Hifnawy, L. (1984). "Effect of Foundation Flexibility on Dynamic Behaviour of Buildings", 8th WCEE, San Francisco, vol. III, 721-728.

Novak, M. and El Sharnouby, B. (1983). "Stiffness Constants of Single Piles", J. Geotech. Eng. Div. ASCE, vol. 109, no. 7, 961-974.

Novak, M. and El Sharnouby, B. (1984). "Evaluation of Dynamic Experiments on Pile Groups", J. Geotech. Eng., June, vol. 110, no. 6, 738-756.

Novak, M. and Grigg, R.F. (1976). "Dynamic Experiments With Small Pile Foundations", Canadian Geotech. J., vol. 13, no. 4, 372-385.

Novak, M. and Han, Y. (1990). "Impedances of Soil Layer With Boundary Zone", J. Geotech. Eng., June, vol. 116, no. 6, 1008-1014.

Novak, M. and Howell, J.F. (1977). "Torsional Vibration of Pile Foundations", J. Geotech. Eng. Div. ASCE, April, vol. 103, no. GT4, 271-285.

Novak, M. and Howell, J.F. (1978). "Dynamic Response of Pile Foundations in Torsion", J. Geotech. Eng. Div. ASCE, vol. 104, no. GT5, 535-552.

Novak, M. and Janes, M. (1989). "Dynamic and Static Response of Pile Groups", Proc. 12th Int. Conf. on Soil Mech. Found. Eng., Rio de Janeiro, August, 1175-1178.

Novak, M. and Mitwally, H. (1990). "Random Response of Offshore Towers With Pile-Soil-Pile Interaction", J. Offshore Mech. and Arctic Eng., Feb., vol. 112, 35-41.

Novak, M. and Nogami, T. (1977). "Soil Pile Interaction in Horizontal Vibration", Earthq. Eng. Struct. Dyn., vol. 5, 263-282.

Novak, M. and Sheta, M. (1980). "Approximate Approach to Contact Problems of Piles", Proc. Geotech. Eng. Div. ASCE National Convention "Dynamic Response of Pile Foundations: Analytical Aspects", Florida, Oct. 30, 53-79.

Novak, M. and Sheta, M. (1982). "Dynamic Response of Piles and Pile Groups", Proc. 2nd Int. Conf. on Numerical Methods in Offshore Piling, University of Texas at Austin, Texas, April, 489-507.

Novak, M., Nogami, T. and Aboul-Ella, F. (1978). "Dynamic Soil Reaction for Plane Strain Case", J. Eng. Mech. Div., ASCE, vol. 104, no. 4, 953-959.

O'Neill, M.W. and Dobry, R. (Eds.) (1980). "Dynamic Response of Pile Foundations: Analytical Aspects", ASCE, 112 p.

Ostadan, F. (1983). "Dynamic Analysis of Soil-Pile-Structure Systems", Ph.D. Thesis, U. of California, Berkeley, Grad. Div.

Pak, R.Y. and Jennings, P.C. (1987). "Elastodynamic Response of Pile Under Transverse Excitations", J. Eng. Mech. Div., ASCE, vol. 113, 1101-1116.

Parmelee, R.A., Penzien, J., Scheffey, C.F., Seed, H.B. and Thiers, G.R. (1964). "Seismic Effects on Structures Supported on Piles Extending Through Deep Sensitive Clays", Inst. Eng. Res., U. of California, Berkeley, Rep. SESM 64-2.

Penzien, J. (1970). "Soil-Pile Foundation Interaction", in Earthquake Engineering, Ed. R.L. Wiegel, Prentice-Hall, Englewood Cliffs, New Jersey, 349-381.

Penzien, J., Scheffey, C.F. and Parmelee, R.A. (1964). "Seismic Analysis of Bridges on Long Piles", J. Eng. Mech. Div. ASCE, no. EM3, 223-254.

Poulos, H.G. (1968). "Analysis of Settlement of Pile Groups", Geotechnique, vol. 18, 449-471.

Poulos, H.G. (1971). "Behaviour of Laterally Loaded Piles. II - Pile Groups", J. Soil Mech. Foundations Div. ASCE, vol. 97, no. SM5, 733-751.

Poulos, H.G. (1979). "Group Factors for Pile-Deflection Estimation", J. Geotech. Eng. Div. ASCE, no. GT12, 1489-1509.

Poulos, H. (1980). "An Approach for the Analysis of Offshore Pile Groups", Proc. Num. Methods in Offshore Piling, ICE, London, 119-126.

Poulos, H.G. and Davis, E.H. (1980). Pile Foundations Analysis and Design, John Wiley & Sons, 397 p.

Prakash, S.F. and Chandrasekaran, V. (1977). "Free Vibration Characteristics of Piles", Proc. 9th Int. Conf. Soil Mech. Found. Eng., Tokyo, vol. 2, 333-336.

Prakash, S. and Sharma, H.D. (1990). "Pile Foundations in Engineering Practice", Wiley, Chap. 7.

Rajapakse, R.K.N.D. and Shah, A.H. (1987a). "On the Longitudinal Harmonic Motion of an Elastic Bar Embedded in an Elastic Half Space", Int. J. Solids Structures, vol. 23, 267-285.

Rajapakse, R.K.N.D. and Shah, A.H. (1987b). "On the Lateral Harmonic Motion of an Elastic Bar Embedded in an Elastic Half-Space", Int. J. Solids Structures, vol. 23, 287-303.

Rajapakse, R.K.N.D. and Shah, A.H. (1989). "Impedance Curves for an Elastic Pile", Soil Dyn. and Earthq. Eng., vol. 8, no. 3, 145-152.

Roesset, J.M. (1980). "Stiffness and Damping Coefficients of Foundations", Proc. of Session on Dyn. Response of Pile Found.: Analytical Aspects, ASCE National Convention, Florida, Oct., 1-30.

Roesset, J.M. and Angelides, D. (1979). "Dynamic Stiffness of Piles: Numerical Methods in Offshore Piling", Inst. of Civil Engrs., London, 75-82.

Roesset, J.M., Stokoe, K.H., Baka, J.E. and Kwok, S.T. (1986). "Dynamic Response of Vertically Loaded Small-Scale Piles in Sand", Proc. 8th European Conf. Earthq. Eng., Lisbon, vol. 2, 5.6/65-72.

Sanchez-Salinero, I. (1982). "Static and Dynamic Stiffnesses of Single Piles", Geotech. Eng. Rep. GR82-31, Civil Eng. Dept., U. of Texas at Austin, August, 225 p.

Sen, R., Davies, T.G. and Banerjee, P.K. (1985). "Dynamic Analysis of Piles and Pile Groups Embedded in Homogeneous Soils", Earthq. Eng. Struct. Dyn., vol. 13, 53-65.

Sen, R., Kausel, E. and Banerjee, P.K. (1985). "Dynamic Analysis of Piles and Pile Groups in Non-Homogeneous Soils", Int. J. Num. and Anal. Meth. in Geomech., vol. 9, 507-524.

Sheta, M. and Novak, M. (1982). "Vertical Vibration of Pile Groups", J. Geotech. Eng. Div. ASCE, April, vol. 108, no. GT4, 570-590.

Shimizu, N., Yamamoto, S. and Koori, Y. (1977). "Three-Dimensional Dynamic Analysis of Soil-Structure System by Thin Layer Element Method", Trans. Archit. Inst. of Japan, March, no. 253, 31-41.

Sooysmith, C. (1896). "Concerning Foundations for Heavy Buildings in New York City", Trans. ASCE, Vol. 35, 459-469.

Swane, I.C. and Poulos, H.G. (1982). "A Theoretical Study of the Cyclic Shakedown of Laterally Loaded Piles", Res. Rep. No. R415, Univ. of Sydney, School of Civil and Mining Eng., July.

Tajimi, H. (1966;1969). "Earthquake Response of Foundation Structures", Rep. of Fac. Sci. Eng., Nihon University, 1966.3, 1.1-3.5 (in Japanese). See also Tajimi, H. "Dynamic Analysis of a Structure Embedded in an Elastic Stratum", Proc. 4th WCEE, Chile.

Tajimi, H. (1977). "Seismic Effects on Piles", Proc. of Specialty Session 10, State-of-the-Art Rep., 9th ICSMFE, Tokyo, July 14, 15-26.

Tajimi, H. and Shimomura, Y. (1976). "Dynamic Analysis of Soil-Structure Interaction by Thin Layered Element Method", Trans. Archit. Inst. of Japan, May, No. 243, 41-51 (in Japanese). Also: Rep. "Dynamic Analysis of a Single Pile Embedded in Horizontally Layered Soils", College of Sci. & Tech., Nihon Univ., Tokyo.

Takemiya, H. and Yamada, Y. (1981). "Layered Soil-Pile-Structure Dynamic Interaction", Earthq. Eng. and Structural Dyn., vol. 9, 437-458.

Terzaghi, K. and Peck, R.B. (1967). "Soil Mechanics in Engineering Practice", 2nd Ed., New York: Wiley.

Trochanis, A.M., Bielak, J. and Christiano, P. (1988). "A Three-Dimensional Nonlinear Study of Piles Leading to the Development of a Simplified Model", Res. Rep. R-88-176, Dept. of Civil Eng., Carnegie Mellon Univ., Pittsburgh, PA, 175 p.

Waas, G. and Hartmann, H.G. (1981). "Pile Foundations Subjected to Dynamic Horizontal Loads", European Simulation Meeting "Modelling and Simulation of Large Scale Structural Systems", Capri, Italy, Sept., (also SMIRT, Paris), 17 p.

Waas, G. and Hartmann, H.G. (1984). "Seismic Analysis of Pile Foundations Including Pile-Soil-Pile Interaction", Proc. 8WCEE, San Francisco, vol. V, 55-62.

Wolf, J.P. (1985). "Dynamic Soil-Structure Interaction", Prentice-Hall Inc.

Wolf, J.P. and von Arx, G.A. (1978). "Impedance Functions of a Group of Vertical Piles", Proc. ASCE Specialty Conf. on Earthq. Eng. & Soil Dyn., Pasadena, CA, vol. II, 1024-1041.

Wolf, J.P. and von Arx, G.A. (1982). "Horizontally Travelling Waves in a Group of Piles Taking Pile-Soil-Pile Interaction Into Account", Earthq. Eng. & Struct. Dyn., vol. 10, no. 2, 225-237.

Wolf, J.P. and Weber, B. (1986). "Approximate Dynamic Stiffness of Embedded Foundation Based on Independent Thin layers With Separation of Soil", Proc. 8th European Conf. on Earthq. Eng., Lisbon, vol. 2, 5.6/33-5.6/40.

Wolf, J.P., von Arx, G.A., de Barros, F.C.P. and Kakubo, M. (1981). "Seismic Analysis of the Pile Foundation of the Reactor Building on the NPP Angra 2", Nuclear Eng. & Design, vol. 65, no. 3, 329-341.

Woods, R.D. (1984). "Lateral Interaction Between Soil and Pile", Proc. Int. Symp. on Dyn. Soil-Structure Interaction, Minneapolis, MN, Sept., 47-54.

Yan, L. (1990). "Hydraulic Gradient Similitude Method for Geotechnical Modelling Tests With Emphasis on Laterally Loaded Piles", Ph.D. Thesis, Fac. of Grad. Studies, Univ. of British Columbia, Canada.

Papers Presented to This Conference (1991)

Ahmad, S. "Seismic response of floating piles to obliquely incident waves".

Arya, A. and Arya, A.S. "Pile group stiffness for seismic soil-structure interaction".

Baba, K. "Dynamic analysis of soil-piles interaction systems under earthquake type loading".

Kagawa, T. "Seismic response of axially loaded pile group".

Kobori, T., Nakazawa, M., Hijikata, K., Kobayashi, Y., Miura, K., Miyamoto, Y. and Moroi, T. "Study on dynamic characteristics of a pile group foundation".

Nogami, T., Jones, H.W. and Mosher, R.L. "Seismic response analysis of pile supported structure: Assessment of commonly used approximations".

Nomura, S., Tokimatsu, K. and Shamoto, Y. "Soil-pile-structure interaction during liquefaction".

Otani, J., Nogami, T. and Konagai, K. "Non-linear time domain numerical model for pile group under transient dynamic forces".

Purkayastha, R.D. and Dey, S. "Behaviour of cyclically loaded model piles in soft clay".

Wu, S., Chen, Y., Cai, Y. and Chen, L. "Analysis of pile-soil dynamic interaction by combination of BEM and FEM".